The Redistributive Effects of Monetary Policy in an Overlapping Generation Model[1]

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Abstract

I study the redistributive effects of monetary policy in a full-fledged Overlapping Generations New Keynesian model with financial frictions. The model, calibrated to match the U.S. demographic structure and with a marginal propensity to consume and inequality among households, can reproduce negative redistribution elasticities as presented in Auclert (2019). However, I find that the effect of monetary policy is weaker than a standard New Keynesian model would imply. Unlike in a New Keynesian model, a weaker indirect general equilibrium effect induces a smaller increase in wages, and thus savers are found to bear a negative wealth effect from a lower interest rate.

JEL classification: D31, E12, E21, E44, E52

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1 Introduction

Analyses of the effect of monetary policy through the redistribution channel have attracted increasing attention in recent years. Monetary policy may have redistributive effects because a decrease in interest rates reduces the debt burden of borrowers while causing a loss in interest income for savers. However, it is commonly argued that such monetary policy may ultimately benefit savers as well because of the powerful general equilibrium channel of the monetary policy. According to this view, decreasing interest rates should create strong demand from borrowers that leads to a general equilibrium increase in labor demand. This generates greater labor income for savers, which offsets losses in their interest income. In fact, this view is consistent with the implications of standard models such as the simple or two-agent New Keynesian model.

I revisit this argument in a full-fledged overlapping generation New Keynesian (OLGNK) model, which pays special attention to age heterogeneity. The model is rich in three regards. First, because it adopts an OLG economy, it captures households consumption and income profiles over the full life cycle (Gourinchas and Parker 2002). The model assumes that households live from age 21 to 80 and work from age 25 to 65. Household labor productivity shows systematic age variations over the life cycle, and those households are subject to labor productivity shocks. Second, the model captures the U.S. demographic structure. Household survival probabilities are taken from the Mortality Tables of the Internal Revenue Service, and the resulting distribution of the population by age is similar to that of the U.S.. Third, the marginal propensity to consume (MPC) implied by the model is consistent with empirical findings for MPC (e.g., Johnson, Parker, and Souleles 2006; Parker, Souleles, Johnson, and McClelland 2013; Broda and Parker 2014; Misra and Surico 2014).

In order to ensure the model implies empirically realistic MPC, I incorporate financial frictions into the model. The financial frictions in this paper also address the criticism of representative New Keynesian (NK) models that monetary policy operates almost exclusively through the direct intertemporal substitution channel. In contrast to this implication of standard NK models, micro survey data show that there is a significant proportion of hand-to-mouth households whose consumption is insensitive to changes in interest rates. Incorporating this aspect into a model, Kaplan, Moll, and Violante (2018) show that the indirect channel, such as a general equilibrium response of wages, is far more important than the direct/intertemporal substitution channel.

The nature of financial frictions in this paper differs from models in which households face a fixed borrowing limit (e.g., Bewley-Huggett-Aiyagari models). In particular, I suppose that in each cohort, there is some proportion of households that face borrowing constraints. A key assumption here is that the borrowing constraint constrains borrowers' issuance of *new* debt. The constrained households face high borrowing costs and those costs increase with the amount of new borrowing.[1] Because they have a higher marginal propensity to consume, the general equilibrium channel of monetary policy which works through wages will become far more important than the intertemporal substitution channel. In my setting, the degree of the financial frictions can be parameterized to match an empirical MPC. Also, the model nests special cases such as hand-to-mouth households and households without any borrowing constraints.

In order to isolate the effects of some model parameters and facilitate model comparisons, I present results of a first experiment comparing a homogeneous OLGNK model and a OLGNK model without any financial frictions. The homogeneous OLGNK model, in which household labor productivity is constant and age-independent, serves as a benchmark because aggregate outcomes from that model are almost identical to those from a representative agent New Keynesian model. Therefore, this comparison answers the question: If age-dependent labor productivity is set to match expected income profiles over the life cycle in the US economy, how much of a difference does this make (relative to the homogeneous economy) to the model's prediction for the response of the economy to a monetary shock? In the homogeneous OLGNK model households hold no assets, whereas in the OLGNK model households accumulate assets according to life cycle considerations.

I find that the effects of a monetary policy shock are substantially smaller in the OLGNK model compared to the homogeneous OLGNK model. In the homogeneous OLGNK model, output always stays above the steady-state level in response to a transitory decrease in the nominal interest rate. The monetary policy shock induces a initial increase in aggregate demand and labor income. This increase is large enough so that households can consume and save more today, which also raises consumption in the future. In contrast, in the OLGNK model, there is a smaller initial increase in households' demand, which is eventually countered with a decrease in household demand after the first few periods. A transitory monetary shock leads to endogenous persistence through movements in wealth distribution. The weak initial response implies that some generations do not gain as much from the monetary policy as other generations.

[1]This setup can be understood as one-side debt adjustment costs. In the real world, a large amount of new borrowing would decrease credit scores. This may increase borrowing rates endogenously, and so may be costly for borrowers. This financial frictions assumption differs from a basic borrowing constraint with a fixed borrowing limit.

To understand the differential effects of monetary policy on different generations, I calculate how a monetary policy shock affects the normalized present value of wealth for each generation. My analysis shows that households of age over 47 lose from a negative monetary policy shock and households close to retirement are the biggest losers, whereas the monetary policy shock tends to benefit younger generations. This is not the case for the homogeneous OLGNK model, in which the monetary policy shock increases the present value of wealth for all generations. In this case, the effect of monetary policy via the general equilibrium channel is powerful; a large increase in wages induced by a strong demand with a negative monetary policy shock allows households to consume more today without sacrificing any consumption tomorrow. However, in the OLGNK model, due to the negative wealth effect that savers (older generations) must bear, a negative monetary policy shock induces a rather moderate increase in aggregate borrowing. This leads to a rather subdued general equilibrium effect on wages, and these increases in wages cannot offset the negative wealth effect of a cut in interest rates that savers have to bear.

However, without financial frictions, the OLGNK model neither captures empirical MPC nor fully reflects the inequality among households observed in the data. In the second experiment, I calibrate the OLGNK model with financial frictions (OLGNK-FF) to match empirical MPC and the consumption inequality observed in the data. I then address the question of whether the redistributive channel amplifies the effect of monetary policy on aggregate output. Auclert (2019) argues that monetary policy may be amplified through the redistribution channels such as interest-rate exposure, the Fisher channel, and earnings heterogeneity. He presents the redistribution elasticities associated with those channels as sufficient statistics to gauge the redistributive effects of monetary policy. To compare my results with those of Auclert, I also calculate these redistribution elasticities from model simulations.

I show that the redistribution elasticities in the OLGNK-FF model are also negative, as is the case with empirical and model statistics in Auclert's study. However, the computed model impulse responses show that the redistribution channels still do not amplify the effects of monetary policy on aggregate output. Despite the presence of the financial frictions, the aggregate responses are weaker in some cases. These results contrast with Auclert's in that they suggest that the redistributive effect of monetary policy may not be captured by the redistribution elasticities. The main reason for the weaker response to monetary policy is that, unlike the case in a Bewley-Huggett-Aiyagari model, an initial increase in borrowing from the constrained household is limited due to the financial frictions. Therefore, an initial increase in demand is subdued, which translates into a smaller increase in wages. Consequently, older generations would still have to bear a larger negative wealth effect. This leads monetary policy to yield a smaller effect on aggregate output in the OLGNK-FF model compared to the OLGNK model.

In addition, this result suggests that the effects of monetary policy through the redistribution channels may depend heavily on the nature of the financial frictions. In a Bewley-Huggett-Aiyagari model with a fixed borrowing limit, households that are close to their borrowing limit may respond strongly to a negative monetary policy shock by increasing their amount of borrowing. This strong increase in demand generates a powerful general equilibrium effect through wages which alleviates the burdens savers would bear. In my model, households are constrained on new borrowing. Because borrowing by the constrained younger generations is not as sensitive to changes in interest rates, this specification leads to weaker general equilibrium effects. As a result, the type of the financial frictions considered here turns out to mute the equilibrium dynamics of output in response to a monetary shock.

Related Literature My paper is related to the literature on two-agent dynamic general equilibrium models with nominal rigidities such as Curdia and Woodford (2010, 2016), Debortoli and Galí (2018), and Bilbiie (2019a, 2019b), and builds on the findings of Curdia and Woodford. The stochastic evolution of the types of household in my paper is similar to theirs; the key difference is that in my paper the types of household are categorized based on borrowing constraints into constrained and unconstrained households, instead of into borrowers and savers.

There is a growing literature studying the effects of monetary policy in the presence of agent heterogeneity (e.g., McKay, Nakamura, and Steinsson 2016; Kaplan, Moll, and Violante 2018; Auclert 2019). Setting aside the nature of financial frictions, the OLGNK-FF model in this paper differs from Bewley-Huggett-Aiyagari models in how it analyzes the redistributive effects of monetary policy in two regards. First, it takes account of systematic age-based variation in income over the life cycle. The empirical evidence from Gourinchas and Parker (2002) suggests that younger generations face upward sloping expected income profiles, whereas middle age households, on the contrary, face declining expected income profiles before retirement. This systematic age-based variation in income translates into systematic variation in asset accumulation across generations. Doepke and Schneider (2006) also illustrate that older households have a large positive net nominal position which makes them more exposed to a cut in interest rates. Second, we can analyze the differential effects of monetary policy on the welfare of households that are in different phases of their working life. One common argument is that we may ignore redistributive effects because they average out in the long run. According to this argument, the same type of households that lose from monetary policy impact may also gain later in their life through a general equilibrium effect. This is the case in a perpetual youth model such as the Blanchard-Yaari which does not allow for retirement or variations in labor income over the life cycle.[2] However, an OLG economy, as modelled in this paper, limits the extent to which older generations, who face downward sloping expected income profiles, gain from the policy.[3]

This paper also contributes to the growing literature on the redistributive effects of monetary policy, which includes the works of Doepke and Schneider (2006), Adam and Zhu (2015), Gornemann, Kuester, and Nakajima (2016), Coibion, Gorodnichenko, Kueng, and Silvia (2017), Kaplan, Moll, and Violante (2018), Auclert (2019) , and Broer, Harbo Hansen, Krusell, and Öberg (2019), among many others. The main difference to such studies is that the present paper develops an overlapping generation model in the presence of nominal rigidities and financial frictions, which is tractable and captures heterogeneity among households.

The remainder of this paper proceeds as follows. Section 2 shows the importance of the nature of financial friction in the evaluation of monetary policy in a simple two-period borrower-saver model. Section 3 builds the OLGNK model with financial frictions and explores how the redistribution channel affects the impact of monetary policy on aggregate outcomes. Section 4 concludes the paper.

2 Simple Two-period Model with Financial Frictions

This section presents a simple two-period borrower-saver model in which there is a constraint on *new* borrowing. I show that the stimulative effects of monetary policy are substantially smaller than a standard model implies and I find that the nature of financial frictions plays a role. An OLGNK model in Section 3 builds upon the insights from this section. A reader more interested in the multi-period dynamic analysis may at this time study equation (7) and skip to Section 3.

[2] In a perpetual youth model the horizon is effectively infinite, although future values are discounted at a higher rate reflecting survival probabilities.

[3] On the other hand, I abstract from the precautionary saving motive under uncertainty. Due to a large state space, I solve models using log linear approximation.

2.1 Environment

The model shares the same approach with Baek (2019), who analyzes the impact of forward guidance in a three-period borrower-saver model. The model includes two periods, $t = 0, 1$, and two types of household: 'savers', denoted S, and 'borrowers', denoted B. The proportions of the population represented by the two types are constant at all times, and equal to $\frac{1}{2}$. Households have preferences

$$u(C_0^i - v(h_0^i)) + \beta^i u(C_1^i - v(h_1^i)) \tag{1}$$

where either $u(c) = \ln c$ or $u(c) = \dfrac{c^{1-\sigma}}{1-\sigma}$ with $\sigma > 0, \sigma \neq 1$; $v', v'' > 0, v(0) = 0$; $\beta < 1$. Households' budget constraints are given by

$$P_0 C_0^i + \frac{B_1^i}{1+i_0} = W_0 \theta_0^i h_0^i + \Pi_0 + B_0^i, \tag{2}$$

$$P_1 C_1^i = W_1 \theta_1^i h_1^i + \Pi_1 + B_1^i \tag{3}$$

where i_0 is a nominal interest rate in period 0; B_1^i denotes holdings of nominal bonds at the end of period 0; $B_0^i = \bar{B}^i$ denotes holding of nominal bonds at the beginning of period 0; θ_t^i is household i's productivity in period t; W_t is the nominal wage; Π_t denotes nominal profits from monopolistically competitive firms.

Households can take negative positions in the nominal bond, but face a (real) borrowing constraint:

$$\frac{B_1^i}{1+i_0} - B_0^i \geq \frac{-P_1 \phi}{1+i_0}. \tag{4}$$

Note that the constraint is on households' *new* borrowing, not on the total amount of debt they have to repay at date 1. This feature is important in relation to the effect of monetary policy.[4]

Households' utility maximization implies that labor supply (regardless of types) is given by

$$\frac{W_t}{P_t} = v(h_t^i). \tag{5}$$

Firms produce output using a linear technology, $Y_t = H_t$, where $H_t = \sum_i \frac{1}{2} \theta_t^i h_t^i$. Following the standard New Keynesian model with sticky prices, firms' demand for labor is determined by the following relationship:

$$\frac{W_t}{P_t} = mc\left(Y_t - Y_t^*\right), \tag{6}$$

[4] Note that without the second term in expression (4), this becomes a standard borrowing constraint, $B_1^i \geq -P_1 \phi$.

where $mc(0) = 1$, $mc_t' > 0$, and Y_t^* is output under flexible prices. For generality, I do not assume a particular model of price setting; rather, I suppose that the inflation function $\pi_1(i_0)$ is known to the monetary authority and the monetary authority chooses the nominal interest rate i_0 at date 0 (and thus the real interest rate $r_0 = \frac{1+i_0}{1+\pi_1}$).

Finally, markets clear:

$$\frac{1}{2}\left(C_t^S + C_t^B\right) = Y_t.$$

An equilibrium is a collection $\{C_t^i, h_t^i, B_1^i, Y_t, W_t, \Pi_1\}_{t=0,1}^{i=S,B}$ such that, given the policy $i_0 \geq 0$ and given initial endowments, households $i = S, B$ choose $\{C_t^i, h_t^i, B_1^i\}_{t=0,1}$ to maximize (1) subject to (2), (3), and (4); firms maximize profits at date 1; and markets clear.

2.2 Equilibrium in a Borrower-Saver Economy

I focus on the case in which the economy is in a recession (e.g., a preference shock) in period 0. This implies that the equilibrium level of output in period 0, Y_0, is less then the flexible-price output Y_0^*. Because I focus on the effects of an interest rate on current output, prices in period 0 are assumed to be fixed and are normalized to unity; firms must hire whatever labor is necessary to meet demand. However, prices are flexible in period 1, $Y_1 = Y_1^*$.[5]

In order to derive the main result as simply as possible, I specialize the model to the following case.

Assumption 1. *Savers are more patient than borrowers, $\beta_S > \beta_B$. Savers have a (weakly) positive endowment of nominal bonds in period 0, $\bar{B}^S = -\bar{B}^B \geq 0$. Borrowers and savers have the same productivity in period 0, $\theta_0^S = \theta_0^B$, but borrowers have a weakly higher productivity in period 1, $\theta_1^B \geq \theta_1^S$. No new borrowing is possible in period 0: $\phi = 0$. Prices are fixed in period 0, which we normalize to unity.*

'Savers' could represent wealthy, older households, that have a large positive net nominal position (Doepke and Schneider 2006), representing nominal bonds and pensions, but low future income. 'Borrowers' represent any households that take the other side of these positions in equilibrium. One interpretation is that borrowers represent households that are young today and will be middle-aged in the future: their negative net nominal position could represent both their fixed-rate mortgage debt, and their tax liabilities, which pay for the savers' pensions (which are fixed in nominal terms).

[5]Note that from equations (5) and (6), $1 \geq v'(h_t)$. The inequality is strict in period 0, but it holds with equality in period 1.

Assumption 1 implies that households have the same income in period 0, $Y_t^S = Y_t^B := Y_t$. The assumption of flexible prices in period 1 implies that income will always be at its efficient level, Y_1^*, in period 1. In light of this, I denote households' (common) level of income in period 0 by Y_0^*. The economy is considered to be in recession in period 0 if $Y_0 < Y_0^*$.

We are interested in equilibria in which borrowers are liquidity constrained and the economy is in recession at date 0. The following lemma describes conditions under which this will be the case. Roughly speaking, these conditions are satisfied if borrowers' discount factor β^B is low and savers' discount factor β^S is high. For the following lemma, I define net consumption, $c_t^i := C_t^i - v(h_t^i)$. Also, note that efficient labor supply h_t^{i*} is determined by $h_t^{i*} := \arg\max_h \theta_t^i h - v(h)$.

Lemma 1. *Suppose parameters satisfy*

$$y_0^{*-\sigma} > \beta^B \left(1 + r_0^*\right) \left[y_1^{B*} - \left(1 + r_0^*\right) \bar{B}^S\right]^{-\sigma}$$

$$y_0^* > \left(\beta^S \left(1 + r_0^*\right)\right)^{-1/\sigma} \left(y_1^{S*} + \left(1 + r_0^*\right) \bar{B}^S\right)$$

where I define

$$y_0^* = \max_h \theta_0 h - v(h),$$

$$y_1^{i*} = \max_h \theta_1^i h - v(h), i = S, B,$$

$$r_0^* > 0.$$

Then if $r_0 = r_0^*$, the equilibrium has $Y_0 < Y_0^*$, where $Y_0^* = \theta_0 h_0^*$.

For proof see the Appendix.

In standard models with nominal rigidities, a lower interest rate can mitigate a current recession, inducing individuals who are on their Euler equations to spend more today. Savers' Euler equations imply that

$$u'\left(c_0^S\right) = \beta^S \left(1 + r_0\right) u'\left(c_1^S\right).$$

If c_1^S is fixed, a decrease in r_0 increases c_0^S. This mechanism assumes that savers' future consumption remains fixed even as the interest rate decreases. This will not be true if households' *new* borrowing is constrained. In this case, an interest rate has a negative wealth effect in the opposite direction to the substitution effect. If this wealth effect is strong enough, it could outweigh the substitution effect.

Proposition 1. *1. Suppose $\sigma = 1$, $\bar{B}^S > 0$, $\theta_1^S = 0$, and parameters satisfy Assumption 1. There exists no equilibrium with $r_0 < r_0^*$ and $Y_0 = Y_0^*$. That is, the central bank cannot restore output to its efficient level by decreasing the real interest rate.*

2. If $\sigma \neq 1$, the elasticity of savers' net consumption with respect to r_0 is

$$\frac{\partial c_0^S}{\partial 1 + r_0} \frac{1 + r_0}{c_0^S} = -\left(\frac{1}{\sigma} - (1 + r_0) \frac{\bar{B}^S}{c_1^S} \right), \tag{7}$$

where

$$c_1^S = y_1^{S*} + (1 + r_0) \bar{B}^S. \tag{8}$$

Proof) 1. The savers' Euler equation states that

$$\frac{1}{c_0^S} = \beta^S (1 + r_0) \frac{1}{c_1^S},$$

and savers' net consumption is $c_1^S = (1 + r_0) \bar{B}^S$. It follows that $c_0^S = \frac{\bar{B}^S}{\beta^S}$, independent of r_0.

2. It follows from the savers' Euler equation:

$$c_0^S = \left(\beta^S (1 + r_0) \right)^{-1/\sigma} [y_1^{S*} + (1 + r_0) \bar{B}^S]. \square$$

In other words, the effect of monetary policy depends on the difference between the savers' intertemporal elasticity of substitution (IES), $\frac{1}{\sigma}$, and the share of nominal bonds in their date 1 consumption (i.e., their exposure to interest rate shocks), $(1 + r_0) \frac{\bar{B}^S}{c_1^S}$. The share of nominal bonds cannot exceed unity; the IES can be greater or less than 1. Proposition 1-1 describes a special case: if the IES and nominal bond shares are both equal to unity, the consumption elasticity is zero.

Proposition 1-2 also highlights that output could potentially even fall in response to a lower interest rate, if the IES is less than 1 and the nominal bond share is large.

Note that a key assumption here is the nature of financial frictions. Due to the financial frictions, borrowers may have limited opportunity to adjust their asset positions, which constrains borrowers' issuance of new debt (see Footnote 2). In the other special case in which every borrower has access to a financial market to adjust positions, the borrowing constraints reduce to ones with a fixed borrowing limit, in which case a lower interest rate would always be expansionary. In this case, when the constraint binds for borrowers, savers' period 1 consumption is

$$c_1^S = y_1^{S*} + \phi, \tag{9}$$

which is fixed, unlike the case in Proposition 1. There is no way that a lower interest rate can reduce savers' period 1 consumption, and the elasticity of the savers' consumption reduces to $\frac{\partial c_0^S}{\partial 1+r_0}\frac{1+r_0}{c_0^S} = -\frac{1}{\sigma}$.

Intuitively, in this economy a decrease in the real interest rate relaxes the borrowers' constraint, and allows them to take on more new debt and spend more today. This stimulates aggregate demand today to induce a large increase in wages that allows savers to raise both their current consumption and savings for future consumption. Therefore, there is no negative wealth effect that savers bear at the end, as can be seen in equation (9). However, if there are constraints on households' new borrowing, an initial increase in aggregate demand of this kind is not large enough to offset a negative wealth effect that hurts savers (equation (8)). This result shows that the nature of borrowing constraints is important when evaluating the effects of monetary policy.

3 A New Keynesian Overlapping Generation Model with Financial Frictions

In an overlapping generation New Keynesian (OLGNK) model, in each period new generations are born and live for a finite period of time, but otherwise, if there exist no financial frictions, the model closely follows a standard New Keynesian model. Here I describe the OLGNK model extended with financial frictions, but the model can be parameterized to nest a case in which there are no financial frictions.

One criticism of the representative NK model is that monetary policy operates almost exclusively through intertemporal substitution, whereas micro survey data show that there is a significant fraction of households who behave as hand-to-mouth consumers (Kaplan, Moll, and Violante 2018). To make the model empirically more realistic, I introduce financially constrained households into the OLGNK model, whose behavior is similar to hand-to-mouth consumers. As they have a higher marginal propensity to consume, the general equilibrium channel of monetary policy which works through wages will be far more important to these households than the intertemporal substitution channel, which is in line with the findings of Kaplan, Moll, and

Violante (2018).[6]

To incorporate financial frictions, I borrow an idea from Curdia and Woodford (2016), where the types of agents follow a Markov process, as explained in the next section.

3.1 Households

Time is discrete, $t = 0, 1, 2, 3, ...$ and the economy lives forever. I use the subscript t to denote the current period. In each period, new households are born, who live for $T + 1$ periods and then die. The age of households born in year s at time t is given by $t - s$. I use the superscript $t - s$ to denote the age of households. I denote by ϕ^{t-s} the probability of surviving until age $t - s$, conditional on being alive at age $t - s - 1$. Then the unconditional probability of survival from age 0 to age $t - s$ is $\bar{\phi}^{t-s} = \Pi_{j=0}^{t-s} \phi^j$. These probabilities define a mass μ^{t-s} of household $t - s$, where the total mass of households is normalized to one, $\sum_{t-s=0}^{T} \mu^{t-s} = 1$.

Each generation consists of the two types of households, where $\tau_t^{t-s}(i) \in \{f, n\}$ indicates household i's type at time t. I use the superscript to denote the type of households.

The net accumulation of savings, N_t, is defined as

$$N_t^{t-s,\tau} = \phi^{t-s+1} B_{t+1}^{t-s,\tau} - A_t^{t-s,\tau},$$

where $B_{t+1}^{t-s,\tau}$ denotes the nominal assets of the type τ household of generation $t - s$ at the beginning of period t and $A_t^{t-s,\tau}$ the nominal assets of the household at the beginning of the period.[7]

Type-f households Households of this type are not financially constrained. They are not subject to the borrowing constraint, except the no Ponzi constraint. Their utility function is given by

$$U_t^{t-s,f} = \sum_{k=t}^{s+T} \beta^k \bar{\phi}^{k-s} v\left(Z^{k-s}\right) \left[\frac{\left(c_k^{k-s,f}\right)^{1-\sigma}}{1-\sigma} - \frac{\left(h_k^{k-s,f}\right)^{1+\psi}}{1+\psi} \right], \tag{10}$$

where $v\left(Z^{t-s}\right) = 1 + \bar{v} Z^{t-s}$ which is an age-dependent preference shifter that reflects deterministic household characteristics Z^{t-s} (e.g., family size), $c_t^{t-s,f}$ is the consumption of the type f household of generation $t - s$ at time t, and $h_t^{t-s,f}$ is labor supply.

[6] The effects of the monetary shock on aggregate consumption can be decomposed into direct and indirect components, $dc_t = \underbrace{\frac{\partial c_t}{\partial i_t} di_t}_{\text{direct effect}} + \underbrace{\frac{\partial c_t}{\partial W_t} dW_t + \frac{\partial c_t}{\partial D_t} dD_t + \frac{\partial c_t}{\partial G_t} dG_t}_{\text{indirect effects}}.$

[7] Further details are provided in equation (15).

Generation $t-s$ is endowed with labor productivity $\theta^{t-s,f}$ that generates labor income $\frac{W_t}{P_t}\theta^{t-s,f}h_t^{t-s,f}$, where W_t is the nominal wage at time t and P_t is the aggregate price level. Labor productivity is age-dependent, but its distribution is time invariant.

Type-n households Type-n households receive a negative labor productivity shock

$$\theta^{t-s,n} = (1-\xi)\,\theta^{t-s,f}, \tag{11}$$

where $0 < \xi < 1$. This negative income shock positions the type-n households as natural borrowers in the economy.

However, the type-n households are subject to a financial constraint. The type of the financial constraint here is motivated by the insights from Section 2. The financial constraint is imposed into the household's utility in the form of the penality function (identically, liquidity demand) $\varkappa(\cdot)$:

$$U_t^{t-s,n} = \sum_{k=t}^{s+T} \beta^k \bar{\phi}^{k-s} v\left(Z^{k-s}\right)\left[\frac{\left(c_k^{k-s,n}\right)^{1-\sigma}}{1-\sigma} - \frac{\left(h_k^{k-s,n}\right)^{1+\psi}}{1+\psi} + \varkappa\left(n_k^{k-s,n}\right)\right], \tag{12}$$

where $n_t^{t-s,n} = N_t^{t-s,n}/P_t$ and the first order derivative of $\varkappa$ is given by

$$\varkappa'(n) = \exp\left(-\tilde{\varkappa}\cdot n\right) - 1, \text{ where } \tilde{\varkappa} \geq 0. \tag{13}$$

This form of financial frictions is useful in that the degree of financial frictions can be parameterized by choosing $\tilde{\varkappa}$. The model nests two special cases in which financial frictions are irrelevant ($\tilde{\varkappa} = 0$) and households behave as traditional hand-to-mouth consumers ($\tilde{\varkappa} \to \infty$).[8] Also, $\tilde{\varkappa}$ can be chosen so that the MPC of the model is consistent with MPC values from empirical studies.

N_t is the net issuance of nominal debt, which is the household's total borrowing less existing debt that is carried from a previous period. Thus, equation (13) represents a constraint on households' new borrowing, not on the total amount of debt they have to repay at $t+1$. As $\varkappa''(n) < 0$, obtaining new debt, $n < 0$, is costly to the type-n households. Although there is little cost for saving, there is an increasing adjustment cost of raising the amount of debt. Another way to microfound this type of financial frictions would be to use a setting in which only a certain fraction of the constrained households have access to financial markets for further borrowing in

[8] The consumption of the type-n households is characterized as that of traditional hand-to-mouth consumers if $\tilde{\varkappa} \to \infty$ and the size of a negative labor productivity shock ξ is sufficiently large. This leads type-n households to borrow as much as they can to increase consumption in the current period, but the marginal cost of issuing new debt becomes infinity.

each period.[9]

Arguably, borrowing opportunities/constraints would differ depending on employment status and new borrowing would be harder for the unemployed. Also, there are empirical surveys such as the SCE Credit Access Survey (by the Federal Reserve Bank of New York) which could be used as inputs into the model, which opens possibilities for future study.

Another advantage of the model from a technical perspective is that it is not necessary to keep track of the wealth distribution in the economy with a set of assumptions, as described below.

Markov Process for the Evolution of Types and Financial Contracting In order to keep the model tractable, I follow Curdia and Woodford (2016) who develop a two-agent New Keynesian model without the curse of dimensionality.

Each type τ evolves as an independent two-state Markov chain. In particular, at the beginning of each period, a new type for each household of generation $t-s$ is drawn with probability $1-\delta$. When a new type is drawn, the household becomes type-f with probability κ^{t-s} and type-n with probability $1-\kappa^{t-s}$. When a new type is not drawn, which occurs with probability δ, the household remains the same type as in the previous period.[10]

The analysis becomes tractable with an additional form of financial contracting. Thanks to the following form of financial contracting, all households of the same type within the same generation make identical choices, independent of type history. This implies that we can obtain stationary equilibrium without keeping track of the history of the type of individual households.

Households sign state-contingent contracts, which insure against both aggregate and idiosyncratic risks, with one another, but only within the same generation. At the beginning of each period, household i receives the state-contingent transfer, $\tilde{T}_t^{t-s}(i)$, from the insurance agency intermittently. Household i receives the insurance payment only if a new type is drawn for that household, which occurs with probability $1-\delta$; otherwise (with probability δ), households do not have access to the insurance agency and thus, $\tilde{T}_t^{t-s}(i) = 0$. For those households of generation $t-s$ who have access to the insurance agency, post-transfer wealth at the beginning of period t is the same. Those transfers through the insurance agency aggregate to zero each period,

$$\int \tilde{T}_t^{t-s}(i)\, di = 0 \quad \text{for all } t-s. \tag{14}$$

[9]For instance, in each period, the fraction γ of the constrained households do not have an opportunity to obtain more debt, in which case their positions would be equal to their existing position, $B_t^{t-s} = B_{t-1}^{t-s}$.

[10]κ may depend on age, but it is time invariant. Thus, the distribution of the agent's type is stationary.

In Curdia and Woodford (2016), infinitely-lived households expect that they are eventually able to receive transfers from the insurance agency, which equalizes the expected marginal utilities of income across all households of the same type within a generation.[11] In order to obtain a similar result with my model, in which households have finite lifetimes, I suppose that all households have access to the insurance agency with probability 1 at the end of life, $\delta^T = 0$. With this final condition, the aggregation of debt becomes linear, and we obtain stationary equilibrium without keeping track of the wealth distribution in the economy. This simplifies the methodology because it enables the use of local log-linearization methods to characterize the model's solutions.

Budget Constraints The nominal assets of household i of generation $t-s$ at the beginning of any period t, after insurance payments (if any), are given by

$$A_t^{t-s}(i) = (1 + i_{t-1}) B_t^{t-s}(i) [1 - \omega + \omega P_t] + \tilde{T}_t^{t-s}(i), \tag{15}$$

where $B_t^{t-s}(i)$ denotes household holdings of nominal assets at the beginning of period t and $B_t^{T+1} = 0$; i_{t-1} is a nominal interest rate between periods $t-1$ and t; and ω is a constant share of the portfolio invested in real indexed bonds.[12] It is important to note here that the insurance transfer $\tilde{T}_t^{t-s}(i)$ depends only on the history of aggregate and idiosyncratic exogenous states, not on the choice of $B_t^{t-s}(i)$. This implies that households cannot manipulate the amount of the transfer $\tilde{T}_t^{t-s}(i)$ by choosing a lower level of $B_t^{t-s}(i)$.

The end-of-period asset position of a household is given by

$$\phi^{t-s+1} B_{t+1}^{t-s}(i) = A_t^{t-s}(i) - P_t c_t^{t-s}(i) + W_t \theta^{t-s}(i) h_t^{t-s}(i) + D_t^{t-s}(i) + G_t^{t-s}, \tag{16}$$

where $D_t^{t-s}(i)$ denotes the lump-sum distribution of nominal profits from monopolistically competitive firms; and G_t^{t-s} denotes lump-sum transfers from the government. Additionally, I assume that there are annuity markets in which households insure themselves against mortality risk. Specifically, every household continues to keep the assets of the deceased in the same generation. This implies that $B_{t+1}^{t-s}(i)$ is multiplied by ϕ^{t-s+1} in the left side of equation (16).

Optimality Conditions for Households The Euler equation for a generation $t-s$ household

[11] Note that the probability that households do not access to the insurance agency goes to zero, $\lim_{t\to\infty} \delta^t = 0$.

[12] Ideally, households should be able to choose real and nominal assets independently. However, it is well known that this causes indeterminancy because households are indifferent between holding nominal or real assets where there are no arbitrage conditions.

with the current type f and any type history τ^t is given by

$$\lambda_t^{t-s,f} = \beta\phi^{t-s+1}(1-\delta)E_t\left[\frac{(1+i_t)}{\Pi_{t+1}}\tilde{\lambda}_{t+1}^{t-s+1}\right] + \beta\phi^{t-s+1}\delta E_t\left[\frac{(1+i_t)}{\Pi_{t+1}}\lambda_{t+1}^{t-s+1,f}\right], \tag{17}$$

$$\lambda_t^{t-s,f} = v\left(Z^{t-s}\right)\left(c_t^{t-s,f}\right)^{-\sigma}, \qquad \lambda_t^{t-s,n} = v\left(Z^{t-s}\right)\left(c_t^{t-s,n}\right)^{-\sigma}, \tag{18}$$

where $\tilde{\lambda}_{t+1}^{t-s+1} = \kappa^{t-s+1}\lambda_{t+1}^{t-s+1,f} + \left(1-\kappa^{t-s+1}\right)\lambda_{t+1}^{t-s+1,n}$. The Euler equation for a household with the current type n and any type history τ^t is given by

$$\begin{aligned}\lambda_t^{t-s,n} - \varkappa'\left(n_t^{t-s,n}\right) = {} & \beta\phi^{t-s+1}(1-\delta)E_t\left[\frac{(1+i_t)}{\Pi_{t+1}}\tilde{\lambda}_{t+1}^{t-s+1}\right] \\ & + \beta\phi^{t-s+1}\delta E_t\left[\frac{(1+i_t)}{\Pi_{t+1}}\left(\lambda_{t+1}^{t-s+1,n} - \varkappa' n_{t+1}^{t-s+1,n}\right)\right]\end{aligned} \tag{19}$$

Both equations (17) and (19) can be solved forward:

$$\begin{aligned}\lambda_t^{t-s,f} = {} & \sum_{k=0}^{T-1-(t-s)}\beta^{k+1}\bar{\phi}_k(1-\delta)\delta^k E_t\left[\frac{(1+i_{t+k})}{\Pi_{t+1+k}}\tilde{\lambda}_{t+1+k}^{t-s+1+k}\right] \\ & + \beta^{T+1-t-s}\bar{\phi}_{T-(t-s)}\delta^{T-(t-s)}E_t\left[\frac{(1+i_{T+s})}{\Pi_{T+s+1}}\tilde{\lambda}_{T+s+1}^{T+1}\right],\end{aligned} \tag{20}$$

$$\begin{aligned}\lambda_t^{t-s,n} - \varkappa'\left(n_t^{t-s,n}\right) = {} & \sum_{k=0}^{T-1-(t-s)}\beta^{k+1}\bar{\phi}_k(1-\delta)\delta^k E_t\left[\frac{(1+i_{t+k})}{\Pi_{t+1+k}}\tilde{\lambda}_{t+1+k}^{t-s+1+k}\right] \\ & + \beta^{T+1-t-s}\bar{\phi}_{T-(t-s)}\delta^{T-(t-s)}E_t\left[\frac{(1+i_{T+s})}{\Pi_{T+s+1}}\tilde{\lambda}_{T+s+1}^{T+1}\right],\end{aligned} \tag{21}$$

where $\bar{\phi}_k = \Pi_{i=0}^{k}\phi^{t-s+1+i}$.[13]

Equations (20) and (21) tell us that because $\tilde{\lambda}$, which appears in the right-hand side, is independent of the history of the type, $\left\{\lambda_t^{t-s,f}, \lambda_t^{t-s,n}, n_t^{t-s,n}\right\}$ is history independent; all the households in the same generation with the same type choose the same amount of consumption and labor supply in equilibrium.[14] While the type-n households, which suffer from a negative labor productivity shock, are natural borrowers in the economy, the shock is temporary with $\delta < 1$ or they expect the access to the insurance agency eventually.

[13] The last terms in equations (20) and (21) reflect the assumption that all households have access to the insurance agency at the end of their life ($\delta = 0$ at the end of the life cycle).

[14] Equation (16) can be written as $n_t^{t-s,n} = -c_t^{t-s,n} + \frac{W_t}{P_t}\theta^{t-s,n}h_t^{t-s,n} + \frac{D_t^{t-s,n}}{P_t} + \frac{G_t^{t-s}}{P_t}$. If households with the same type choose the same c_t and h_t, then their choice of n_t is also the same. That is, this equilibrium is consistent with equation (21).

A household, which is a wage-taker, supplies hours $h_t^{t-s,\tau}$ that satisfies the labor supply equation:

$$\frac{W_t}{P_t} = \frac{\left(h_t^{t-s,\tau}\right)^{\psi}}{\lambda_t^{t-s,\tau}}. \tag{22}$$

3.2 Standard Part of the Model

Firms A monopolistically competitive firm produces a differentiated good of type j using linear technology

$$Y_t(j) = A_t h_t\left(j\right), \tag{23}$$

where $h_t\left(j\right) \equiv \int h_t\left(j;i\right) di$ is aggregate labor hired of type j; and $h_t\left(j;i\right) = \theta^{t-s,\tau_t(i)} h_t^{t-s,\tau_t(i)}\left(j\right)$ is the efficiency units of labor. Households consume a Dixit-Stiglitz aggregate of differentiated goods, $Y_t = \left[\int_0^1 Y_t\left(j\right)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$, where $\varepsilon > 1$ is the constant elasticity of substitution. This implies that each firm faces the demand schedule:

$$Y_t\left(j\right) = \left(\frac{P_t\left(j\right)}{P_t}\right)^{-\varepsilon} Y_t,$$

and the corresponding price index can be written $P_t = \left[\int_0^1 P_t\left(j\right)^{1-\varepsilon} dj\right]^{1/(1-\varepsilon)}$.

Before addressing the firm's pricing decision, consider its cost minimization problem:

$$\min_{H_t(j)} \frac{W_t}{P_t} h_t\left(j\right) + \varphi_t\left(Y_t\left(j\right) - A_t h_t\left(j\right)\right),$$

where φ_t is a Lagrangian multiplier associated with the production function (23). The first order condition is given by

$$\frac{W_t}{P_t} = \varphi_t A_t. \tag{24}$$

φ_t is thus defined as the firm's real marginal cost.

The firm's pricing setting problem is then to choose $P_t\left(j\right)$ to maximize its profits. An individual firm takes as given the demand curve, the price index P_t, and the wage W_t, and chooses its price P to maximize expected discounted profits. However, each firm has a sticky price, in that it can change the price of its product in a given period with probability $1-\alpha$ (Calvo 1983). This part of the model remains the same as a standard NK model. Solving the firm's price setting decision problem, we obtain the standard Phillips curve (after log-linearization):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{\varphi}_t, \tag{25}$$

where $\kappa = \frac{(1-\varepsilon)(1-\beta\varepsilon)}{\varepsilon}$, $\pi_t = \log \frac{P_t}{P_{t-1}}$, $\hat{\varphi}_t = \log \frac{\varphi_t}{\varphi}$, and $\varphi = \frac{\varepsilon-1}{\varepsilon}$.

In a model in which households are heterogeneous, how firms' profits are distributed across households will affect equilibrium dynamics. As in Kaplan, Moll, and Violante (2018), I assume that profits are distributed in proportion to household productivity each period in a lump-sum manner,

$$D_t^{t-s,\tau_t(i)} = \frac{\theta^{t-s,\tau_t(i)}}{E\left[\theta\right]} \int \left[P_t\left(j\right) - \frac{W_t}{A_t} \right] Y_t\left(j\right) dj.$$

This implies that households' additional income as the profit-sharing component of worker compensation (e.g., dividends, bonuses) is proportional to their labor income.

Monetary Policy Monetary policy is straightforward.

$$1 + i_t = \left(1 + i^*\right) \left(\frac{1 + \pi_t}{1 + \pi^*} \right)^{\phi_\pi} e^{\varepsilon_t^m}, \tag{26}$$

where i^* is a target interest rate and π^* is the central bank's inflation target, which is assumed to be equal to 0.

Fiscal Policy The government budget constraint is given by

$$B_{t+1} = G_t + \left(1 + i_{t-1}\right) B_t. \tag{27}$$

I suppose that the government runs a balanced budget to maintain a stable level of real debt in each period, $\frac{B_{t+1}}{P_t} = \bar{b}$ for all t where $\bar{b}$ denotes (real) government. Then the government budget constraint becomes

$$\bar{b} = \frac{G_t}{P_t} + \left(1 + i_{t-1}\right) \frac{\bar{b}}{1 + \pi_t}. \tag{28}$$

Market Clearing The labor market clears:

$$h_t = \int h_t(j) dj. \tag{29}$$

The goods market clears:

$$\sum_{t-s=0}^{T} \mu^{t-s} c_t^{t-s} \equiv c_t = Y_t = \frac{A_t h_t}{\Delta_t}, \text{ where } c_t^{t-s} = \int c_t^{t-s,\tau_t(i)} di. \tag{30}$$

where $\triangle_t \equiv \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} dj$ is the measure of price dispersion.

The bond market clears:

$$\sum_{t-s=0}^{T} \mu^{t-s} \int \frac{B_{t+1}^{t-s}(i)}{P_t} di = \bar{b}. \tag{31}$$

3.3 The Dynamics of Aggregate Savings

The insurance contract guarantees that all households $t-s$ with access to the insurance agency have identical post-transfer wealth, $\tilde{A}_t^{t-s}$, at the beginning of period t. Integrating equation (15) over all households $t-s$ with access to the insurance agency in period t, the post-transfer wealth (per capita), $\tilde{A}_t^{t-s}$, is given by

$$\tilde{A}_t^{t-s} = P_{t-1}(1+i_{t-1})\, \bar{b}_t^{t-s} \left[1-\omega+\omega P_t\right],$$

where $\bar{b}_t^{t-s} = \int B_t^{t-s-1}(i)\, di$. This equation is derived using equation (14) and the fact that all households have a equal chance to access to the insurance agency, so that the pooling of wealth (per capita) of those households is drawn randomly from among all households. The beginning-of-period wealth of households who do not have access to the insurance agency is instead given by equation (15) with $\tilde{T}_t^{t-s}(i) = 0$.

For the aggregation of individual savings, it is convenient to classify households $t-s$ according to the period in which they last had access to the insurance agency. Let us denote the type of a household who last had access to the insurance agency j periods ago by $\tau(j)$. Aggregating across individual generation $t-s$ households, the aggregate end-of-period assets of the households of type-τ, $B_{t+1}^{t-s,\tau}$, can be written as

$$B_{t+1}^{t-s,\tau} \equiv \sum_{j=0}^{t-s} \kappa^{t-s-j,\tau} (1-\delta)\, \delta^j B_{t+1}^{t-s}(\tau(j)), \tag{32}$$

where $B_{t+1}^{t-s}(\tau(j))$ is given as follows. $B_{t+1}^{t-s}(\tau(0))$, the end-of-period assets of the households that have access to the insurance agency in period t and then draw type τ, will equal

$$\phi^{t-s+1} B_{t+1}^{t-s} \left(\tau\left(0\right)\right) = \tilde{A}_t^{t-s} - P_t c_t^{t-s,\tau} + W_t \theta^{t-s,\tau} h_t^{t-s,\tau} + D_t^{t-s,\tau} + G_t. \tag{33}$$

The right hand side of equation (33) takes into account the fact that the households of the same type within the same generations make the same choices. $B_{t+1}^{t-s}\left(\tau\left(j+1\right)\right)$, the end-of-period assets of the households who have not had access to the insurance agency, is expressed as

$$\phi^{t-s+1} B_{t+1}^{t-s} \left(\tau\left(j+1\right)\right) = B_t^{t-s-1} \left(\tau\left(j\right)\right) - P_t c_t^{t-s,\tau} + W_t \theta^{t-s,\tau} h_t^{t-s,\tau} + D_t^{t-s,\tau} + G_t, \tag{34}$$

for any $j \geq 0$.

According to equation (32), the law of motion for aggregate savings for each type-τ can be obtained by summing equation (33), multiplied by $\kappa^{t-s,\tau}\left(1-\delta\right) P_t^{-1}$, and equation (34) for each value $j \geq 0$, multiplied by $\kappa^{t-s-j,\tau}\left(1-\delta\right)\delta^{j+1} P_t^{-1}$:

$$\begin{aligned} \phi^{t-s+1} b_{t+1}^{t-s,\tau} = {} & \kappa^{t-s,\tau}\left(1-\delta\right)\left(1+i_{t-1}\right)\bar{b}_t^{t-s}\left(\frac{1-\omega}{\Pi_t}+\omega\right) + [\delta\left(1+i_{t-1}\right) b_t^{t-s-1,\tau}\left(\frac{1-\omega}{\Pi_t}+\omega\right) \\ & - \bar{\kappa}^{t-s,\tau}\left(c_t^{t-s,\tau} - \frac{W_t}{P_t}\theta^{t-s,\tau} h_t^{t-s,\tau} - \frac{D_t^{t-s,\tau}+G_t}{P_t}\right)], \end{aligned} \tag{35}$$

where

$$\bar{b}_{t+1}^{t-s} = b_{t+1}^{t-s,f} + b_{t+1}^{t-s,n},$$

$$\kappa^{t-s,f} = \kappa^{t-s},\ \kappa^{t-s,n} = \left(1-\kappa^{t-s}\right),$$

$$\bar{\kappa}^{t-s,\tau} = \bar{\kappa}^{t-s-1,\tau}\delta + \left(1-\delta\right)\kappa^{t-s,\tau},$$

$$\bar{\kappa}^{0,\tau} = \kappa^{0,\tau}.$$

Integrating equation (35) over each type of the households of generation $t-s$, we obtain the expression for the dynamics of aggregate savings for generation $t-s$:

$$\begin{aligned} \phi^{t-s+1}\bar{b}_{t+1}^{t-s} = {} & \left(1+i_{t-1}\right)\bar{b}_t^{t-s}\left(\frac{1-\omega}{\Pi_t}+\omega\right) \\ & - \left(\left(\bar{\kappa}^{t-s} c_t^{t-s,f} + \left(1-\bar{\kappa}^{t-s}\right) c_t^{t-s,n}\right) - W_t\left(\bar{\kappa}^{t-s}\theta^{t-s,f} h_t^{t-s,f} + \left(1-\bar{\kappa}^{t-s}\right)\theta^{t-s,n} h_t^{t-s,n}\right) - \frac{D_t^{t-s,\tau}+G_t}{P_t}\right). \end{aligned} \tag{36}$$

Table 1: Parameter Values for the Model

Parameter	Description	Value
β	time discount factor	0.98
σ	risk aversion	1
$\frac{1}{\psi}$	elasticity of labor supply	1/2
θ	constant elasticity of substitution	10
α	Calvo parameter	0.75
ϕ^{π}	Taylor coefficient	1.5
ω	constant share of indexed bonds	2/3

Thus the dynamics of aggregate savings for each generation depend on the aggregate savings in the previous period, its own past level, and the generation consumption and the various sources of income such as labor income, dividends, and transfers.

The system of equations that describe the model consist of the households' optimality conditions, (17)-(22), the market clearing conditions, (29)-(31), the aggregate-supply relationship, (24) and (25), and the dynamics of aggregate savings, (35)-(36), together with a monetary policy reaction function (26) to specify the nominal interest rate i_t and a fiscal rule (28) to specify the government debt G_t. Those equations suffice to determine the evolution of the variables $\left\{c_t^{t-s,\tau}, h_t^{t-s,\tau}, \lambda_t^{t-s,\tau}, b_{t+1}^{t-s,\tau}, \bar{b}_{t+1}^{t-s}, W_t, \Pi_t, \varphi_t, Y_t, i_t, D_t^{t-s,\tau}, G_t\right\}$.

3.4 Quantitative Results

In this section I numerically analyze the effects of a monetary shock in the calibrated economies.

3.4.1 Calibration

Many of the model's parameters are also parameters of the basic New Keynesian model. These include the discount rate β, the coefficient of risk aversion σ, the inverse of the Frisch elasticity of labor supply ψ, the desired markup of monopolistically competitive firms $\frac{\theta}{\theta-1}$, the probability of maintaining a fixed price α, the coefficient associated with the Taylor rule ϕ^{π}. As Table 1 reports, I choose conventional values for these parameters. I choose the degree of inflation indexation $\omega = 2/3$ in order to take into account the fact that a considerable portion of households' portfolio is invested in real assets. I also conduct robust checks using the values $\omega = 1/3$ and $\omega = 4/5$ and find that they do not affect the conclusion of the paper.

Demographics I suppose each generation lives for 60 periods. In order to take into account the effects of demographic structure on consumption and saving, mortality rates are taken from the Static Mortality Tables of the Internal Revenue Service. The resulting distribution of the population by age, which is reported in Figure 1, is similar to that of the U.S. in 2017.[15][16]

Parameter $\bar{v}$, which is associated with the age-dependent preference shifter, is set to 0.2315, following similar strategies used by Fernández-Villaverde and Krueger (2007) and Attanasio, Kitao, and Violante (2007). The preference shifter is necessary to generate a realistic consumption profile over life cycles.[17] The calibration of the age profile of household characteristics Z^{t-s} is done to match the age profile of consumption, as documented for example in Gourinchas and Parker (2002). They show about a 16 percent rise from the initial value to its peak at age 43, followed by a continuous decline until the end of the life cycle. Consumption at the end of the life cycle is about 60% of the level at its peak. Figure 1 also reports the steady-state consumption across generation in the model.

Labor Productivity I allow for heterogeneity among agents, in a manner, by setting generation-specific labor productivity. By choosing the life-cycle profiles of labor productivity $\{\theta^{t-s}\}$, we can construct realistic life-cycle profiles of income. In Gourinchas and Parker (2002), households income peaks around age 50. Income at age 50 is higher by about 40% than income at age 25, but income at age 65 is lower by about 15% than income at age 50.[18] I choose the generation-specific labor productivity to match such life-cycle profiles of income using spline methods. I assume that households under age 25 do not participate in the labor market, and that households retire at age 65. As no labor income is available for these households, the labor productivity of households outside working life, i.e., those below age 25 or over 65, is set to 0. The expected changes in income create households' desire to smooth consumption over their life cycle, which leads them to accumulate wealth. Figure 1 reports labor productivity over the life cycle. Generation-specific labor productivity θ is normalized by the level of productivity at age 25.

Financial Frictions To compare this model to the model without financial frictions, the new parameters that are needed are those relating to the evolution of types of households and the specification of financial frictions. These parameters include the proportion κ^{age} of households

[15]The age group of 20 to 24 years represents 10% (data) vs 8.8% (model) of the population over 20. For the age group 25-34, the values are 15.8% (data) vs 17.5% (model). For 35-54, 34.2% (data) vs 34.8% (model). For 55-64, 17.1% (data) vs 16.8% (model). For 65+, 21% (data) vs 21.9% (model).

[16]The impact of monetary policy may be different for countries with older populations. In Japan, the age group of 65 years and over accounts for about 34% of the population over age 20.

[17]See also Gourinchas and Parker (2002). Without this preference shifter, consumption is constant over the life-cycle at the steady state.

[18]Auclert (2019) also obtains similar statistics from PSID and SHIW data.

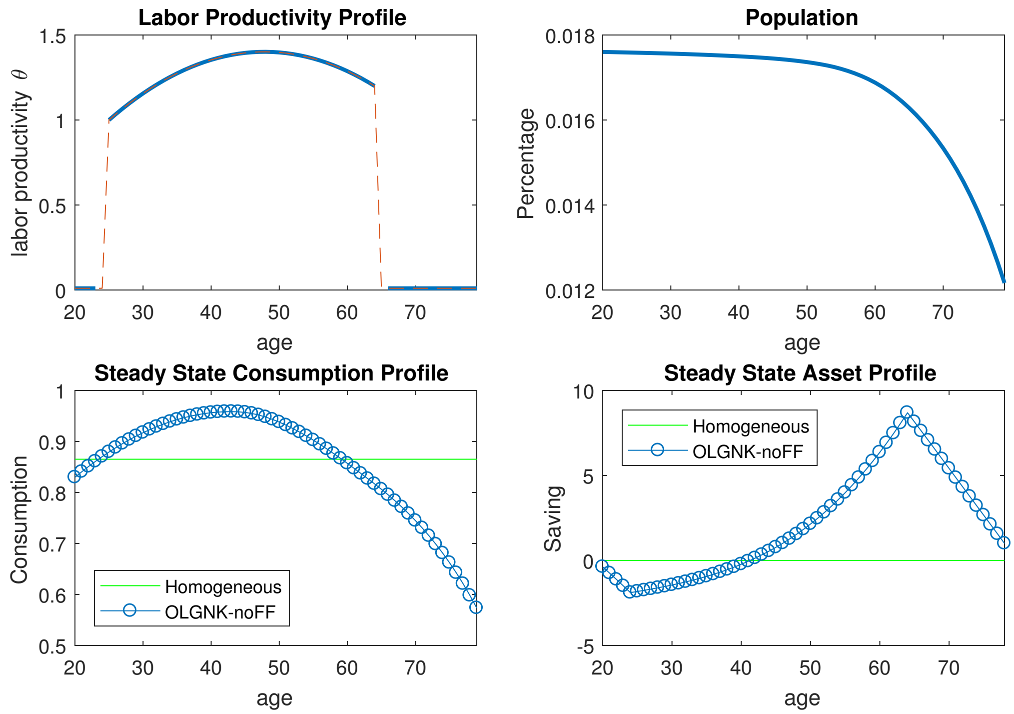

Figure 1: Labor Productivity, Demographic Structure, Steady State Consumption, and Steady State Asset

that are constrained, the degree of financial frictions, $\tilde{\varkappa}$, the magnitude of the labor productivity shock (expression (11)), ξ, and the degree of persistence δ of a household's type.

In the calculation reported here, I assume that $\kappa^{age} = 0.45 - \frac{0.55}{45} \times age$. This implies that the proportion of households that are constrained among the youngest generation (the age of 20) is 0.45 and households become less credit constrained over the life cycle. Gourinchas (2000) and Gourinchas and Parker (2002) show that younger generations are more credit constrained and their marginal propensity to consume is higher than older generations. The choice implies that on average, the proportion of households that are constrained is 0.3. This matches the value in Kaplan, Violante, and Weidner (2014), who report that a reasonable estimate for the proportion of hand-to-mouth consumers in the United States is 0.3.

The other three parameters are chosen to target the following three steady state outcomes. First, the average MPC across all households is 0.24, which is within a resonable range of values reported in the literature (e.g., Johnson, Parker, and Souleles 2006; Parker, Souleles, Johnson, and McClelland 2013; Broda and Parker 2014; Misra and Surico 2014). I also calculate results for an alternative case in which the average MPC equals 0.30, and the results are similar. Second, the variance of log-consumption is equal to 0.26.[19] This number is comparable to 0.25 in Krueger and Perri (2006) and 0.32 in Heathcote, Perri, and Violante (2010). Third, the steady-state level of government debt relative to GDP, $\frac{\bar{b}}{y}$, equals 0.93. This is in line with the debt-to-GDP ratio of the U.S., which has been above 0.9 since 2011.[20]

To obtain these steady state values, I set the degree of financial frictions, $\tilde{\varkappa}$, to 5, I set the magnitude of the labor productivity shock, ξ, to 0.93, and I set the degree of persistence of a household's type, δ, to 0.9.[21] The choice of δ means that the expected time until a household has access to the insurance agency is 10 years. Table 2 reports results for other values of $\tilde{\varkappa}$ for comparison of models.

With those parameter values, the model also replicates some measures of inequality. The Gini coefficient of household consumption at the steady state is 0.22. The 90-10 ratio for the measure of consumption at the steady state is 3.3. Meyer and Sullivan (2013) and Meyer and Sullivan (2017) show that the 90-10 ratio has fluctuated around 4.0 since 1982 in the U.S.. Therefore, the values of these measures in the model are close to their counterparts in real data.

Model Comparisons I compare the main model to a 'Homogeneous' model, a case in which households are homogeneous; all households are financially unconstrained and the household's labor productivity remains the same throughout the life cycle, but the model is otherwise identical. As a result, nominal asset positions are zero across generations at the steady state, and thus we abstract from the redistributive effects of monetary policy. The behavior of households is similar to a standard New Keynesian model. In the 'Homogeneous' model, I set $\theta^{t-s} = 0.6125$ for all generations, because this produces the same steady-state output as the 'OLGNK' model.[22]

I also use a model without financial frictions—the 'OLGNK-noFF' model—for the purpose of comparisons, in which financial frictions are excluded because I set $\tilde{\varkappa} = 0$. The type-n households that receive a negative labor productivity shock can borrow as much as they want in order to smooth their consumption. As the type-n households are not constrained in this model, they face the same kind of the Euler equation (17) as the type-f households face. The model is otherwise identical to the 'OLGNK-FF' model.

[19]The equations for the evolution of wealth can be given in terms of the aggregate for each generation. For this reason, I instead adopt the consumption inequality seen in the data as a measure of inequality.

[20]According to the Worldbank database, the debt-to-GDP ratio of the U.S. has been rising since 2007 and was 0.99 as of 2016.

[21]A lower value of ξ would induce constrained households to borrow less at the steady state, which increases households' aggregate savings, $\bar{b}$. For $\xi = 0.8$, $\frac{\bar{b}}{y}$ equals 0.42. However, the qualitative results are very similar.

[22]As is consistent with a standard New Keynesian model, impluse responses to a monetary shock in the 'Homogeneous' model do not depend on a specific choice of θ.

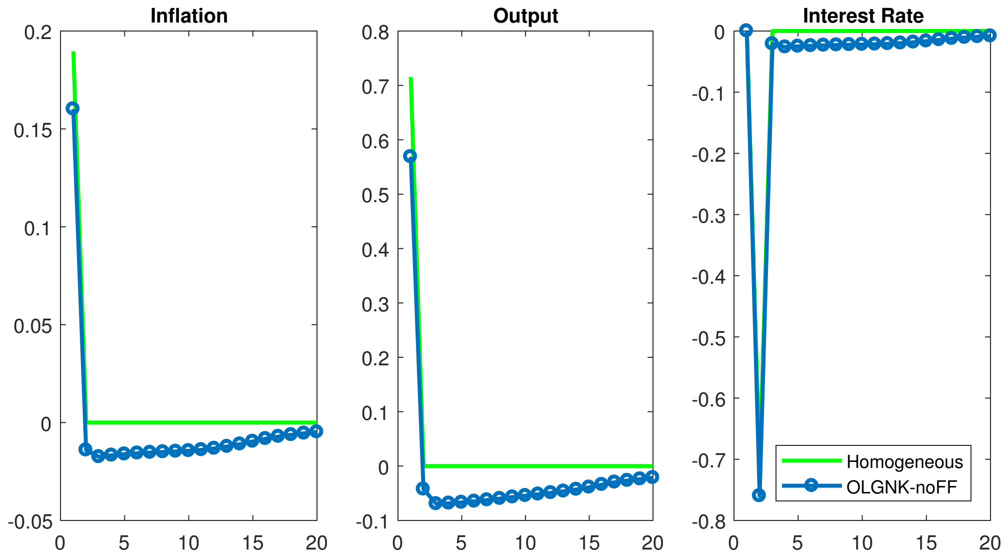

Figure 2: Temporary Nominal Interest Rate Cut: Homogeneous vs OLGNK-NoFF

In numerical exercises, I log-linearize all OLGNK models around the same steady state for comparison purposes.[23][24]

3.4.2 Numerical Results: Homogeneous and OLGNK-noFF Models

In order to understand how the age-productivity profile θ^{t-s} affects the predicted responses to a monetary shock, I begin by comparing the 'Homogeneous' and 'OLGNK-noFF' models. Comparison of these cases shows the difference made by household asset positions across generations.

Impulse Response to a Monetary Shock I consider the equilibrium responses to a temporary monetary policy shock, represented by a unit decrease in ε_t^m. Figure 2 shows the impulse responses of output and inflation along with the nominal interest rate. An expansionary monetary shock generates a boom accompanied with increased inflation. However, the impact of the shock on output and inflation is smaller in the 'OLGNK-noFF' model than the other model, especially in the first few periods. The immediate impact of the shock on output is lower by about 20% in the 'OLGNK-noFF' model, and output is below the steady state level in the next few periods.

The reason for such different responses is that in the 'Homogeneous' model, households

[23] Although the 'Homogeneous' model has a different steady state to the other models, its first order dynamic responses only depend on the common parameters such as β, σ, and α.

[24] To simulate the model for a different value of $\tilde{\varkappa}$ at the same steady state, I add a constant to expression (13). Results are presented in Table 2.

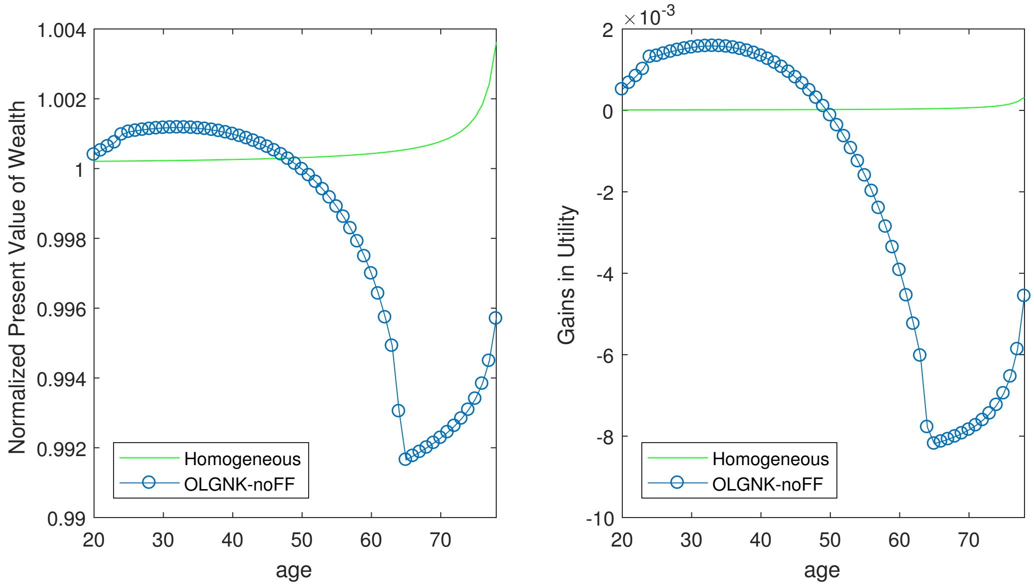

Figure 3: Redistributive Effects Across Generations

immediately increase demand (through the intertemporal substitution channel) to the extent to which the higher wages induced by an increase in demand allow them to raise consumption today without sacrificing any consumption in the future. This is possible because households increase borrowing and saving simultaneously to offset each other, thanks to the substantial increase in wages which occurs through the general equilibrium channel.

Figure 1 indicates that households' asset positions across generations play a role in producing my numerical results. In the 'Homogeneous' model, households are identical and thus their asset position is zero, which is similar to the standard NK model. In the 'OLGNK' model, households are borrowers when they are younger. But as consumption-smoothing households accumulate wealth for retirement, older generations hold positive asset positions.

Redistributive Effects of Monetary Policy Heterogeneity in households asset positions affects equilibrium dynamics. Even though a decrease in the interest rate raises the present value of wealth at time t, the purchasing power of some generations may decrease. This occurs because a lower interest rate also raises the present value of consumption at time t (equivalently, the price of consumption). This implies that it is necessary to adjust the present value of wealth using the price of consumption in order to correctly evaluate the impact of an interest rate cut on the value of household wealth.

To this end, I compute gains and losses for generations as a percentage deviation of the normalized present value of wealth from the steady state values throughout the rest of their life. The present value of wealth is normalized by the present value of consumption at the steady state, which is discounted by a new time path of $\left\{\frac{1}{1+i_k}\right\}_{k=t}^{\infty}$ after a monetary shock at time t.

Figure 3 shows the nominal asset holding profile and the redistributive effects of monetary policy by contrasting the two models. In the 'Homogeneous' model, households gain universally across generations from a monetary shock. Because the shock is transitory, older generations gain more in terms of the present value of wealth than younger generations. The redistributive effect is prominent in the 'OLGNK-noFF' model, in which different generations hold different asset positions. Noticeably, the monetary shock has a negative impact on the present value of wealth of older households. Older households (over the age of 51) tend to lose more, whereas younger households gain more from the monetary shock. In particular, a lower (real) interest rate accompanied by higher inflation hurts middle generations who are closer to retirement. As they carry a large amount of savings for retirement, the wealth of these households is exposed directly to the lower interest rate, but wages do not rise sufficiently to offset this negative wealth effect of the monetary policy due to the subdued increase in aggregate demand. In contrast, households around the age of 35, who are borrowers, benefit most from a lower real interest rate.

In Figure 3, I also compute gains and losses for generations as a (cumulative/average) deviation of utility from the steady state values throughout the rest of their lives. The average gains (or losses) are computed by dividing cumulative gains by remaining years of life for each respective generation. In the 'Homogeneous' model, in which the equilibrium responses are expected to be similar to a standard NK model, all generations benefit from the monetary expansion and there is no distinctive redistribution across generations in this model. In terms of the average gains in utility, older generations benefit more than younger generations, because the boom is short-lived. In the 'OLGNK-noFF' model, the redistributive effects are noticeable, as older generations lose whereas gains accrue to younger generations. This is because older generations, who are savers, lose from higher inflation and a lower interest rate whereas younger generations, who are borrowers, gain from them.

3.4.3 Numerical Results: OLGNK-FF and OLGNK-noFF Models

Like a standard NK model, traditional interest channels are the main transmission mechanism of monetary policy in the OLGNK model without financial frictions. Also, the MPC among households is about 1/60, which is much smaller than the values empirical studies suggest. In

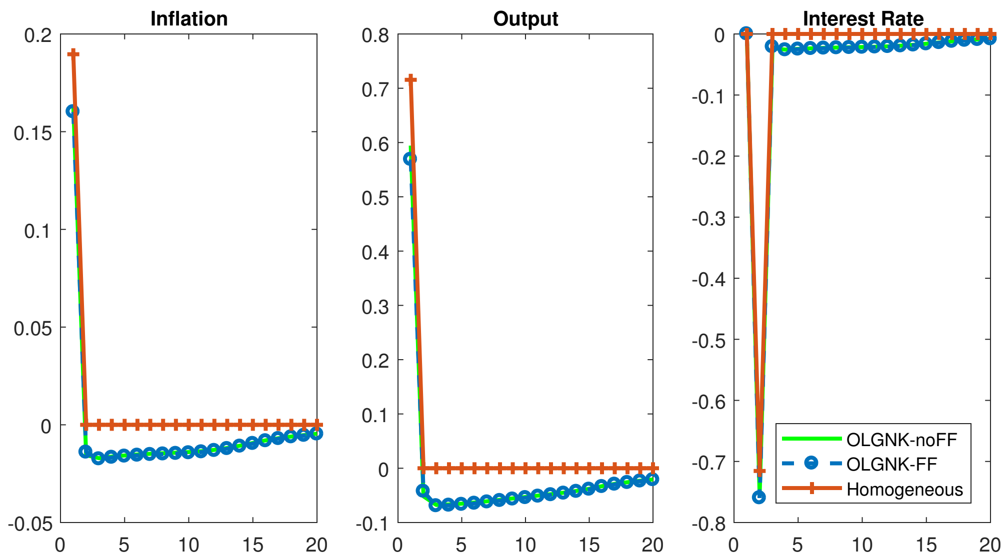

Figure 4: Temporary Nominal Interest Rate Cut: OLGNK-FF vs OLGNK-noFF

this section, I address these issues by assuming that there exist constrained households with $\tilde{\varkappa} > 0$ and study how this assumption affects the previous results.

Impulse Response to a Monetary Shock The main result in this section is that the financial frictions I introduce in this study do not amplify equilibrium dynamics in response to a monetary policy shock. Indeed, equilibrium dynamic responses are muted in the model with financial frictions. To illustrate this, I consider a monetary policy experiment using the 'OLGNK-FF' and 'OLGNK-noFF' models.

Figure 4 shows the impulse responses of output and inflation to a temporary monetary policy shock. The initial increases in output and inflation in the 'OLGNK-FF' model are slightly lower compared to those in the 'OLGNK-noFF' model.[25] The effect of a monetary shock on aggregate output is smaller in an economy with financial frictions. As the type-n households cannot take on debt to take advantage of a lower interest rate, aggregate demand does not respond as much, which leads to a smaller rise in wages. In both models, the initial boom is followed by a slump for longer periods before tracking back to the steady state.

Figure 5 presents the impulse responses of output and inflation to a persistent monetary

[25]Quantitatively, the initial response of output depends on how monopoly profits are distributed to the households. In the case in which monopoly profits are equally distributed across households, the initial increases in inflation and output are still smaller in the 'OLGNK-FF' model compared to the 'OLGNK-noFF' model. Other results are also similar.

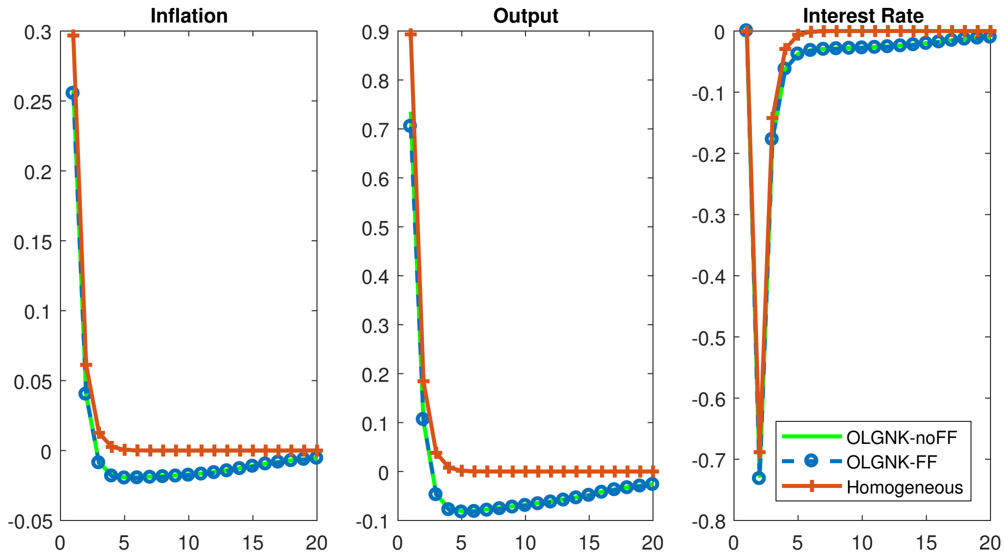

Figure 5: Persistent Nominal Interest Rate Cut ($\rho = 0.3$): OLGNK vs OLGNK-noFF

policy shock ($\rho = 0.3$), which shows that the qualitative results do not change even when a monetary policy shock lasts a few periods.

This result contrasts with the response of aggregate output in the simple Two-Agent New Keynesian (TANK) economy as shown in Kaplan, Moll, and Violante (2018). The aggregate outcome in TANK is the same as in the representative agent NK model because a decreased direct response to the interest rate is offset by an increased indirect general equilibrium effect through wages.

Relationship to Auclert (2019) Auclert (2019) categorizes the redistributive effects into an earnings heterogeneity channel, a Fisher channel, and an interest rate exposure channel. To evaluate these effects, he proposes sufficient statistics, such as the redistribution elasticities of consumption for real interest rates, prices, and income, which may be used to estimate the redistributive effects of monetary policy on aggregate outcomes. Table 2 calculates these redistribution statistics to evaluate whether they help us to predict the redistributive effects of monetary policy in the OLG framework. The table reports two sets of statistics. One set (labeled "model") is calculated numerically from direct model simulations. The other set (those labeled "Auclert" or redistribution elasticity) consists of the redistribution elasticities proposed by Auclert, calculated using the equilibrium dynamics of output from model simulations.[26]

[26] $M = E\left(MPC_i \frac{Y_i}{E[c_i]}\right)$, $\Xi_Y = Cov\left(MPC_i, \frac{Y_i}{E[c_i]}\right)$, $\Xi_P = Cov\left(MPC_i, \frac{NNP_i}{E[c_i]}\right)$, $\Xi_R = Cov\left(MPC_i, \frac{URE_i}{E[c_i]}\right)$, $S = E\left[(1 - MPC_i)\frac{c_i}{E[c_i]}\right]$.

Table 2: Consumption and Redistribution Elasticities (Temporary Shock)

	OLG-noFF	$\tilde{\varkappa} = 1$	$\tilde{\varkappa} = 3$	$\tilde{\varkappa} = 15$
Aggregate Consumption (model)	0.60%	0.59%	0.57%	0.55%
Consumption_unconstrained (model)	0.56%	0.56%	0.56%	0.56%
Consumption_constrained (model)	0.04%	0.03%	0.01%	-0.01%
Aggregate Consumption (Auclert)	0.48%	0.62%	1.30%	1.83%
Consumption_unconstrained (Auclert)	0.38%	0.38%	0.39%	0.39%
Consumption_constrained (Auclert)	0.11%	0.24%	0.91%	1.45%
M (Income-weighted MPC)	0.0333	0.0351	0.0440	0.0511
Ξ_Y(Redistribution elasticity for Y)	-0.0302	-0.0563	-0.1909	-0.3022
Ξ_P(Redistribution elasticity for P)	0.0907	0.0223	-0.3255	-0.6096
Ξ_R(Redistribution elasticity for R)	0.2389	0.0240	-1.0696	-1.9631
S (Substitution channel)	0.9322	0.9202	0.8587	0.8080
$E\left[MPC_i\right]$	0.0647	0.0931	0.2393	0.3599

As can be seen in Table 2, the signs of redistribution elasticities for income Ξ_Y, prices Ξ_P, and real interest rates Ξ_R, obtained from model simulations are all negative in the presence of financial frictions $\tilde{\varkappa} > 0$. This result is consistent with empirical statistics in Auclert.[27] Based upon the signs of those empirical statistics, which are also all negative in his study, he argues that the redistributive effects of monetary policy are likely to amplify the effects of monetary policy.

Even though the signs of all these statistics are also negative in the OLG models with financial frictions, the redistributive channels of monetary policy still do not amplify its effect on aggregate outcomes. One important difference between my model and a model with a fixed borrowing limit (e.g, $b_{t+1} \geq -\phi$) is related to the way high MPC households initially respond to a monetary policy shock. In a model with a fixed borrowing limit, households that are close to their borrowing limit strongly respond to a monetary policy shock by increasing the amount of borrowing. A sizeable increase in demand induces a large increase in wages, which provides sufficient benefits to savers to offset any negative income effects from a decrease in real interest rates. Thus the effect of monetary policy may be amplified via this general equilibrium channel.

In contrast, in the OLGNK-FF model, the borrowing of the constrained households (high MPC households) is not as responsive to a monetary policy shock because the issuance of new debt is costly for those households. This implies that increases in aggregate demand and wages are smaller and older generations have to bear the negative wealth effect of a monetary policy shock, which decreases the net present value of the consumption of those households. This leads

[27] See Table 4 in Auclert (2019).

Table 3: Consumption and Redistribution Elasticities (Persistent Shock, $\rho = 0.3$)

	OLG-noFF	$\tilde{\varkappa} = 1$	$\tilde{\varkappa} = 5$	$\tilde{\varkappa} = 15$
Aggregate Consumption (model)	0.74%	0.73%	0.71%	0.69%
Consumption_unconstrained (model)	0.69%	0.69%	0.70%	0.70%
Consumption_constrained (model)	0.05%	0.04%	0.01%	-0.01%
Aggregate Consumption (Auclert)	0.49%	0.64%	1.37%	1.94%
Consumption_unconstrained (Auclert)	0.39%	0.39%	0.40%	0.40%
Consumption_constrained (Auclert)	0.10%	0.25%	0.98%	1.54%
M (Income-weighted MPC)	0.0333	0.0351	0.0440	0.0511
Ξ_Y(Redistribution elasticity for Y)	-0.0302	-0.0563	-0.1909	-0.3022
Ξ_P(Redistribution elasticity for P)	0.0907	0.0223	-0.3255	-0.6096
Ξ_R(Redistribution elasticity for R)	0.2389	0.0240	-1.0696	-1.9631
S (Substitution channel)	0.9322	0.9202	0.8587	0.8080
$E\left[MPC_i\right]$	0.0647	0.0931	0.2393	0.3599

to a further decrease in the responsiveness of aggregate demand to a negative monetary policy shock.[28]

Also, my models can handle persistent dynamics of the economy. As can be seen in the Figures, even a purely transitory one-time shock leads to endogenous persistence through the redistribution of wealth. However, Auclert's sufficient statistics approach does not apply to such a case. Table 3 reports the same set of statistics for a persistent monetary policy shock, which shows the same qualitative results as discussed so far.

4 Conclusion

I build an OLGNK model in which some households are constrained on the issuance of new debt. The model captures demographic structure and inequality among households in the U.S.. The nature of financial frictions in my model is different to that in a Bewley-Huggett-Aiyagari model with a fixed borrowing limit. Although the redistribution elasticities computed from the numerical simulations of the OLGNK-FF model are consistent with empirical statistics, monetary policy has significantly smaller effects on aggregate output than in a standard NK model. This result suggests that the nature of financial frictions is important in evaluating the redistributive effects of monetary policy. One direction for future research is to better understand the differences in how different kinds of financial frictions affect results and evaluate which are more important.

[28]The redistributive effects across generations remain similar as in Figure 3.

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Appendix : proofs

Proof of Lemma 1

Suppose these parameter restrictions are satisfied. We construct an equilibrium with $i_0 = i_0^*$, as follows. Let $\bar{\pi}_1 := \pi_1(0, i_0^*)$ denote the resulting rate of inflation when $i_0 = i_0^*$, and r_0^* the corresponding real interest rate. From the households' labor supply,

$$h_t^i = h_t, \text{ for } i = S, B, \ t = 0, 1.$$

Since prices are flexible at date 1, equilibrium at date 1 is efficient, $Y_1 = Y_1^*$, and thus

$$v(h_0) = \bar{A}mc(Y_0 - Y_0^*), \ v(h_1) = \bar{A}.$$

$$h_1 = h_1^* := \arg\max_h \bar{A}\theta_1^i h - \theta_1^i v(h), \text{ for } i = S, B. \tag{37}$$

Since $\theta_t = \theta_t^S = \theta_t^B$ for $t = 0, 1$, profit functions are given by

$$\Pi_t = P_t \bar{A}\theta_t h_t - W_t \theta_t h_t, \text{ for } t = 0, \text{ and } \Pi_1 = 0,$$

and net income becomes

$$y_t^i := \frac{W_t}{P_t}\theta_t^i h_t + \frac{\Pi_t}{P_t} - \theta_t^i v(h_t) = \bar{A}\theta_t^i h_t - \theta_t^i v(h_t), \tag{38}$$

$$y_0^i = y_0, \quad \text{for } i = S, B,$$
$$y_1^i = y_1^{i*}, \text{ for } i = S, B.$$

The asset market clearing implies that

$$B_t^S = -B_t^B, \tag{39}$$

$$B_t^i = \bar{B}_t^i, \text{ for } i = S, B, \ t = 1.$$

Define the net consumption as $c_t^i := C_t^i - \theta_t^i v(h_t)$. Plugging (38) and (39) into the budget constraints,

$$c_0^i = y_0^i, \ c_1^i = y_1^i + \frac{\bar{B}^i}{P_1} \text{ for } i = S, B.$$

Using the savers' Euler equation which is given by

$$y_0^{-\sigma} = \beta^S \left(1 + r_0^*\right) \left(y_1 + \left(1 + r_0^*\right) \bar{B}^S\right)^{-\sigma}, \tag{40}$$

we construct y_0 and y_1 as follows:

$$y_1 = y_1^*,$$
$$y_0 = \left(\beta^S \left(1 + r_0^*\right)\right)^{-1/\sigma} \left(y_1^{S*} + \left(1 + r_0^*\right) \bar{B}^S\right).$$

Note that given y_0 and y_1, h_0 and h_1 are determined by (37) and (38).

From equation (40) and the second inequality in the lemma, output is less than its efficient level at date 0, $y_0 < y_0^*$.

It remains to check that the borrowers' Euler equation with a strict inequality. That is:

$$y_0^{-\sigma} > \beta^B \left(1 + r_0^*\right) \left(y_1^{B*} - \left(1 + r_0^*\right) \bar{B}^S\right)^{-\sigma},$$

where we use the fact that $\bar{B}^B = -\bar{B}^S$. This inequality is satisfied given the first inequality in the lemma and by the fact that $y_0 < y_0^*$.□