What Doesn't Kill You Makes You Riskier: The Impacts of CBDC on Banking Stability*

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March 13, 2023

Abstract

We investigate the impact of central bank digital currency (CBDC) on banking stability associated with its programmbility. Banks will find costly to hold onto its depositors: as baseline consumer benefit from using CBDC (denoted by "R") increases, more consumers will adopt CBDC despite a high deposit rate, downsizing the banks' balance sheets. Particularly, highly financially intelligent consumers will prefer new CBDC-based financial services, weakening market discipline in banking by the remaining depositors. To overcome the high borrowing cost, the banks take on excessive risks when R has intermediate values. Moreover, the aggregate surplus from banking is U-shaped in R when the banking instability gets underway.

JEL classification: D80, E58, G18, G21, G53

Keywords: CBDC, programmability, smart contracts, banks' risk-taking, financial stability

*We are grateful to William Cong, Mohammad Davoodalhosseini, Rodney Garratt, Sabrina Howell, Kee-Youn Kang, Young Sik Kim, Ohik Kwon, Heyng Keun Koo, Jongsub Lee, Kuan-Hui Lee, Michael Lee, Alfred Lehar, Jaevin Park, Raghavendra Rau, Harald Uhlig and seminar participants at the Bank of Korea, University of Calgary, and Ajou University for their excellent comments and discussion on the paper.

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1 Introduction

Attention to Central Bank Digital Currency (CBDC) has rapidly grown among policy makers, bankers, and customers due to the two factors: the declining trend of physical money usage and the fast-growing IT technology including fintech (blockchain) and cryptocurrecy (*private* digital currency). Central banks in many countries published numerous policy reports on the optimal design of CBDC architecture.[1] One of the important policy issues commonly found in these reports is how to prevent CBDCs from an unintended consequence of disintermediation and financial instability. Probably the most discussed financial stability issue related to the disintermediation in the literature is that CBDC, due to its superiority in payment and settlement with its degree depending on the level of the CBDC interest rate, can easily replace conventional bank money and thus dry up the bank deposit pool (e.g., Chiu et al. (forthcoming), Keister and Sanches (2022), Kim and Kwon (2022), Whited, Wu and Xiao (2022), and reference therein).

In this paper, we analyze an impact of CBDC on financial (in-)stability associated with banks' risk-taking, related to the important but unexplored feature of CBDC in the literature: *programmability*. To better understand our research question, it is worth noting that depositors will have two different types of investment vehicles after CBDC is launched: conventional bank deposits for *indirect* investments and CBDC for *direct* investments as well as for convenience in payments and settlements. Specifically, the programmability implies that financial consumers can have direct access to new innovative and automated services through smart contracts built on CBDC. For example, individual CBDC holders, particularly those with high financial intelligence, can directly trade tokenized houses or parcels of land via CBDC-based smart contracts, instead of indirectly funding (lending to) homeowners through financial intermediaries in exchange of deposit interests. As these new services become more accessible for CBDC users, CBDC will quickly replace the conventional bank deposits regardless of whether it is interest-bearing or not, which will eventually influence the risk structure in the banking sector. Namely, banks may have an incentive to take on excessive risks in search

[1] See, for example, Benos, Garratt and Gurrola-Perez (2017), Bindseil (2020), Chapman et al. (2017), Choi et al. (2021), and Kumhof and Noone (2018).

for high yields, in response to the increased cost of borrowing caused by the competition with CBDC-based financial service providers.

To address our research question, we build a principal-agent model in banking that involves two policy issues on CBDC – financial inclusion and market structure in banking sector – and how they can influence risk-taking behavior of a bank. The model consists of a bank and a continuum of financial consumers with a unit measure, where each financial consumer is endowed with one unit of capital. The bank raises funds by offering a deposit contract with the same repayment to the financial consumers. The bank then invests the funds in a constant-return-to-scale financial project that gives binary stochastic returns. There are two types of financial projects, safe and risky ones. The risky project yields a strictly higher return than the safe project does when both projects give a positive return *ex post*, while the safe project efficiently yields a higher ex-ante expected return. Since the bank takes the residual return after full repayment of its debts, the bank possibly prefers investing in the risky project when its cost of borrowing is very high, which creates a moral hazard problem.

Each financial consumer chooses either depositing her wealth into the bank or purchasing CBDC. A critical assumption in our model is that consumers are heterogeneous in their financial intelligence, which determines their preferences on various types of money. Specifically, each financial consumer has her own type $\theta \in [0, 1]$. We further assume that the financial consumer's surplus from CBDC is θR, which strictly increases with θ: the more financially intelligent the consumer is, the better can she make use of a new type of currency. In the real world, the size of R, representing the baseline consumer surplus from CBDC, is largely determined by the *infrastructure provision* and *financial regulation* on the retail CBDC usage (see Section 2).[2] Instead, if the consumers choose the conventional bank money, they get repaid conditional on that the bank's project yields a high return. These "depositors" can collectively monitor whether or not the bank chooses the inefficient risky project for its own sake. If the bank is detected to choose the risky project, the depositors can forcefully request the bank to switch its investment back to the safe project. However, the probability to detect

[2]We will also discuss more detailed background about the CBDC-related infrastructure and regulations in the beginning of Sections 5.

the bank's excessive risk-taking depends on the average level of financial intelligence of the depositors.

Our main finding is that the bank becomes financially weak by taking on excessive risk if and only if R is neither too high nor too low. By the standard single-crossing property, there exists a unique threshold $\hat{\theta}$ such that the financial consumer with type θ chooses CBDC if and only if $\theta > \hat{\theta}$. It is then straightforward that $\hat{\theta}$ decreases with R: more financial consumers adopt CBDC as a new means of investment and settlement as the consumer surplus from CBDC increases. A lower $\hat{\theta}$ also means that the remaining bank depositors are less financially intelligent on average, thereby weakening the market discipline in banking. To maintain the size of its balance sheet, the bank has to offer a generous terms of the deposit contract to make its depositors stick with the traditional bank money.

Interestingly, the bank's risk-taking behavior does not monotonically vary with R, while the customers' choice does. First, if R is sufficiently low, the bank does not have to offer a substantially high deposit rate to hold onto its depositors. Since the cost of borrowing is relatively low, the bank does not have to make a risk-shifting decision onto the risky project.[3] Second, if R is very high, the bank has to offer an extremely high deposit rate to maintain the size of its deposit pool. However, such a strategy is unprofitable due to the resulting high cost of borrowing. Instead, the bank would rather allow a large fraction of depositors to switch to CBDC, and therefore, scale down its balance sheet. Such a strategy substantively decreases the cost of borrowing, so the bank has no incentive for excessive risk-taking.

We find that the bank takes on excessive risk when R has intermediate value. In this case, the consumers' benefit from CBDC is relatively high, so the bank must offer a sufficiently generous deposit contract to the consumers in order to manage its investment in a large scale. Such an investment strategy obviously increases the repayment cost, which adversely provides the bank with an incentive to switch its project to the risky one: given the high cost of deposit repayment, excessive risk-taking yields a high net expected return to the bank; the remaining depositors are those with relatively low levels of financial intelligence, so

[3] In addition, there is a second-order effect from disciplining the bank: the remaining depositors have high monitoring ability, on average, which makes the bank difficult to conceal its risk-taking.

they cannot effectively monitor the bank's risk-taking behavior.

We further study how the aggregate surplus from banking varies with R when the banking sector becomes financially weak due to its excessive risk-taking (i.e., when R has intermediate value). Our theory predicts that when the banking instability gets underway, the aggregate surplus from banking decreases with R for relatively small values of R and increases with R for relatively large values of R. Namely, as R increases, the probability of undertaking the risky project increases (the negative effect on the aggregate surplus), but a larger fraction of consumers switch to CBDC, limiting the total size of the bank's risky investment (the positive effect). We find that the negative effect is stronger (weaker) than the positive effect for relatively small (large) R. This result provides novel insight into a government policy when the banking instability takes place as a result of the introduction of CBDC. Namely, when the underlying consumer benefit of CBDC unavoidably grows over time thanks to continuous market-driven innovations in fintech technology, it may be socially desirable to promote CBDC-based digital financial services even further from a banking stability perspective: a wide adoption of CBDC by financial consumers can stem the flow of funds into risky investments made by conventional banks.

Our paper is closely related to the growing literature on the optimal design of CBDC such as Agur, Ari and Dell'Ariccia (2022), Andolfatto (2021), Chiu et al. (forthcoming), Choi et al. (2021), Chiu and Davoodalhosseini (2021), Fernández-Villaverde et al. (2021), Davoodalhosseini (2022), Garratt and Zhu (2020), Keister and Sanches (2022), Keister and Monnet (2022), Kim and Kwon (2022), Panetta (2022), Williamson (2022), and Whited, Wu and Xiao (2022). The important feature that distinguishes CBDC from physical cash is that central banks can directly provide interests to CBDC holders, which creates additional flexibility in monetary policies since central banks can control both inflation policies (on cash) and interest rate polices (on CBDC). By the same token, however, the interest-bearing feature leads to inevitable competition between CBDC and the conventional bank deposits, and thus can create various financial instability problems including bank runs or maturity mismatches. Therefore, the focus on this literature is by and large to investigate the implications resulting from financial consumers' choice between CBDC and conventional bank money, and then the optimal design of CBDC by making more cash-like or deposit-like. Some other related issues discussed in the literature include privacy protection (or consumers' preference for privacy), network effects, and governance issues of private information from digitalization.

Compared to this literature, our main contribution is to shed light on unnoticed factors that cause the financial instability problem: societal environments and governmental provisions/regulations when the retail CBDCs are used as direct investment vehicles among financially intelligent customers. Note that most of the literature argues that well-designed CBDC will not threaten the banking stability or suggests proper designs of CBDC to address the related issues (Ahnert et al., 2022). Our motivation comes from a different view that regardless of the CBDC design (either cash-like or deposit-like), disintermediation and, more importantly, banking instability will take place after the adoption of CBDC as the baseline benefit generated from its programmability or legitimate usage of smart contracts built on CBDC eventually grow up to a certain level. Our analysis also provides important implications on how the degree of banking (in-)stability resulting from the mass-adoption of CBDC is actually shaped by the consumers' benefit from retail usage of CBDC. Note that as the baseline benefit from CBDC increases, credit supply shrinks and thus lending generally decreases, which can usually be considered to be inefficient in the literature. However, we find that as the benefit increases, there is a positive effect of preventing inefficient use of capital from banks' high risk-taking.

Our paper is also related to the financial literacy literature in general (Campbell et al. (2011), Lusardi and Mitchell (2014), Rutledge (2010) and references therein) This literature has documented large heterogeneity in consumer behaviors and has investigated implications for economic welfare and related policy implications. We contribute to this literature by showing that the consumer heterogeneity in financial intelligence can have significant implications on banking stability associated with the adoption of new digital currency.

The remaining paper proceeds as follows. Section 2 provides detailed background for the motivation of our research regarding the CBDC design and its programmability. Section 3 lays out the model. The equilibrium analysis and its implications are investigated in Section 4 and Section 5, respectively. Section 6 provides concluding remarks. All proofs are relegated to Appendix.

2 Background and Motivation

In this section, we explain the current status of CBDC around the world and describe why infrastructure provisions and financial regulations are important in designing CBDC related to its programmability.

2.1 Current Status of CBDC around the World

According to the CBDC tracker by the Atlantic Council, 119 countries (representing over 95 percent of global GDP) are exploring a CBDC as of March, 2023.[4] There are 11 countries including Bahama, Jamaica, and Nigeria that already launched CBDC. Note that the financial system in these countries were not well developed so the introduction of digital money is expected to bring a huge benefit into the system by leaping several development stages.

Is the similar positive benefit expected to be brought for other countries by CBDC? People in developed and liberal societies do not find much inconvenience in the existing financial system; They would not be willing to take the potential risk caused by the issue of privacy protection and the fact that the database of the entire financial system can be breached, implying that CBDC might be a single point of failure. As summarized before, a major consensus from the existing literature is that a properly designed CBDC will not lead to disintermediation, nor financial instability. However, this result from the academic literature is not sufficient to derive policy makers in developed countries to consider its real implementation. Central bankers and policy makers tend to be on the negative side rather than on the positive side (see, e.g., Anthony and Michel (2023) and references in it). A representative example is the recent speech by Governor Christopher J. Waller of the U.S. Federal Reserve System in October, 2022[5]: CBDC is unlikely to provide arrangement for international

[4] See https://www.atlanticcouncil.org/cbdctracker/)

[5] See https://www.federalreserve.gov/newsevents/speech/waller20221014a.htm.

transactions better than cash (current U.D. dollars) does now and to reshape the liquidity and depth of the U.S. capital market.

We believe that CBDC will eventually appear all round the world as many others do. However, as usual in history a new technology adoption takes time. Its arrival could be delayed due to the uncertainty and the potential risk generated by the adoption. More specifically, given that policy makers are skeptical, now CBDC proponents should present *something new* that have not been feasible under the current financial system, but will provide great benefit to the financial system.

2.2 New Benefit from CBDC: Progammability

What would such new benefits that can be obtained from CBDC? As many have emphasized the superiority of the programmable money, we view that the *legitimate* usage of smart contracts built on CBDC can bring completely new benefit to the financial customers. To facilitate the discussion, let us consider the following example of the liquidity creation process by using the smart contracts in a Defi application:

Example 1. *Suppose you have 20 Etherum tokens. Ethereum is $1500 as of March 2023. The total amount of net worth is $30,000.*

1. *Put all the Ethereum holdings into a Vault in MakerDao.*
2. *Then, you can borrow $30,000/1.5 = $20,000 worth of stablecoins called Dai as the current collateralization ratio is 1.5.*
3. *Spend $20,000 to buy Ethereum at Uniswap.*
4. *Repeat 1 to 3 again for infinite times.*

In the end, you can create $\frac{30{,}000}{1-1/1.5}$ *= $90,000.*

Surprisingly the liquidity creation process shown in Example 1 is the same as the money creation process in the current banking system (i.e., the fractional reserve banking system). It is even much faster thanks to automation. Not only using deposit contracts in this way, banks also create credit through collateralization or securitization of their assets such as mortgage contracts. Similar processes are feasible and executable by using Defi platforms such as AAVE, Compound, MakerDao, and so on.

Example 1 demonstrates that now it is possible for individual financial customers or retail customers to be able to execute what only specialists such as large institutions or investment banks have done so far. In this sense, the blockchain technology indeed opens up new opportunities to retail investors who can take advantage of the programmability or smart contracts.

2.3 Challenges

However, a fundamental problem of the liquidity creation process in Defi is that one can view it as nothing but a Ponzi scheme if he/she considers that Ethereum, i.e., the underlying asset, in Example 1 has no value like physical or real assets such as housing and land. Hence, smart contracts used in the current blockchain space are unlikely to be "generally" adopted by the public in a near future since they only use crypto-assets as underlying assets.

What about using real asset tokenization in the case of Example 1? More specifically, if the underlying asset in Example 1 is a tokenized real asset such as a house instead of Ethereum, it is no more a Ponzi scheme and the general public will not be too reluctant to take advantage of these Fintech applications.

Note that real asset tokenization using smart contracts is already technically feasible. For example, it was 2018 in which there were the first use cases such as Aspen Coin and Manhattan Condo tokenization.[6] Then, why is that its actual use cases have been rare sinc then? The main reason is that asset tokenization by using current blockchain technology is highly complicated and costly: it requires various validation steps including legal documentation through lawyers, accountants, realtors, and investment bankers until a valid tokenized

[6] The link for Aspen coin is https://aspencoin.io/; The link for the case of the first Manhattan condo tokenization is https://www.forbes.com/sites/rachelwolfson/2018/10/03/a-first-for-manhattan-30m-real-estate-property-tokenized-with-blockchain/?sh=68d64e7a4895.

asset ownership is put on a blockchain ledger.

This complication originates from the fact that registry services such as the land title ownership transfer requires third parties to check out the validity of the transaction. In other words, real asset tokenization by using current blockchain technology does not fully take advantage of the superiority of the programmability.

2.4 Infrastructure Provision and Financial Regulations

What do we need for tokenizing real assets such as housing and land without much costs and complications? The first requirement is a proper infrastructure: the registry data should be stored on blockchain. Second, even if a government provides registry service on blockchain, another challenge is whether the financial regulation allows individual customers to write a smart contract to tokenize his/her real properties. If so, to what extent? Is it only allowed to trade the entire ownership? What about a fractional ownership trade? If so, how can token holders with a fractional ownership of a condo share cash flows generated from renting the condo? A lot of questions about the ownership and cash flow management can arise.

First, regarding the infrastructure, it is hardly believed that a government will put the registry data on private permissionless blockchains such as Ethereum and ADA.[7] Most of governments in the world will develop its own blockchain for property ownership data services. Government-regulated data services can apply to various different types of ledgers including the ledgers for the stock, bond, and derivative ownership, national or local governmental health records, and asset positions in numerious over-the-counter markets. Second, regarding the financial regulation, what matters is the level of *interoperability* between the CBDC ledger and the corresponding real asset ownership registry ledger. For example, who can or cannot write and call a smart contract on the CBDC blockchain so as to transfer the ownership and to reach a settlement on the land title ledger? Can only a large financial institution do so? Or can a realtor or an individual customer do so? These questions fall into the legal area.

From the above examples, it is intuitive that better the infrastructure provisions or

[7] Here private does not means private blockchain, but means non-government regulated blockchain.

more favorable financial regulations will increase the benefits from using the programmability of CBDC. Without seeing a significant benefit called the 'baseline benefit' from CBDC in our model, developed countries would not introduce CBDC (we will further elaborate the baseline benefit when we described the model in Section 3).

In summary, 'to what extent such baseline benefit is provided' is an important question that should and will be addressed at the architecture design stage of CBDC for its real implementation. How it will be determined will necessarily affect financial stability. This point and related police implications have been largely unexplored so far, which is the motivation of this paper.

3 Model

Consider an economy of a bank and a continuum of lenders with measure 1, both of whom are risk-neutral. The lenders are endowed with one unit of capital. In this economy, the bank borrows from the lenders to undertake projects by offering a deposit contract that specifies the terms of repayment. Each lender, after receiving the bank's offer, decides whether to deposit her money into the bank or to exchange her money for digital currency issued by a central bank. After raising the capital, the bank selects from among different types of projects. The lenders monitor whether or not the bank chooses the project type unfavorable to the lenders. When the game ends, the project gives a return that is accrued to the lenders and the bank according to the deposit contract.

Bank's project selection: We assume that the bank can undertake a constant-return-to-scale financial project via external financing. Specifically, the bank offers a deposit contract that repays each lender $D > 0$ at the end of the game in return for borrowing one unit of capital. The bank can initiate the financial project labelled $i \in \{G, B\}$ by investing one unit of capital per unit. The type-G project is safe and yields a fixed return S with probability 1, where the type-B project is risky and yields a positive return H with probability p and zero return with probability $1 - p$. We assume that the bank's project selection cannot be contractible. That is, there is a potential moral hazard problem between the bank and the lenders, which will be specified below.

Lenders' lending decision: Lenders in the economy make their best lending decisions based on their knowledge of the given financial system. Specifically, we assume that the lenders are uniformly distributed over $[0, 1]$ and each lender is indexed by $\theta \in [0, 1]$, where θ is henceforth referred to as a "type" of lender and represents the lender's financial intelligence. In particular, θ captures a lender's capability of taking advantage of the digital economy derived from the introduction of CBDC. We will further elaborate on this aspect when we explain the model implications later.

There are two options for each lender: depositing her money in the bank or holding the digital currency. If a lender can choose to deposit her money in the bank, she becomes a "depositor." On the other hand, if a type-θ lender exchanges her wealth for digital currency, its expected return is $R(\theta)$, an increasing function of θ, implying that the benefit from using the CBDC account increases with the level of the lender's financial intelligence. For simplicity, we assume the constant returns to scale technology for the return, i.e., $R(\theta) = \theta R$, where constant $R > 0$ represents the level of the baseline surplus from using CBDCs, which is universal to everyone. The return is assumed to be leveraged by the level of the lender's financial intelligence.

As mentioned in Section 2, two different factors broadly affect the value of R: IT infrastructure and regulatory policies regarding CBDC implementation. For example, customers enjoy a convenience yield or additional utility from transacting CBDCs since it provides fast settlement services and low fees for domestic and international transactions. They can also earn financial gains by trading tokenized real or digital assets and by creating their own digital properties through CBDC wallets. These activities will be feasible through smart contracts built on CBDC. Currently, customers can undertake theses activities by using smart contracts and digital wallets provided by cryptocurrencies or DeFi (Decentralized Finance) platforms. However, we do not believe that the conventional banking sector will lose a significant portion of customers against the DeFi sector since the accessibility to DeFi is limited to a fairly small group of tech-oriented people. Rather, these new opportunities will be open to the entire public when CBDCs are introduced in earnest. Given that, it is important to note that these benefits will be larger as better IT infrastructure for CBDC transactions is provided and more favorable regulation is imposed. For example, the tokenization of real assets or personal properties will not be feasible unless registry services for land/housing/car titles are available through a distributed ledger designed to be interoperable with the CBDC ledger. In this case, the level of the interoperability depends on the extent to which the infrastructure for such transitions between the two ledgers is provided and how favorably the financial regulation is toward it. Similarly, how much international transactions will be executed through CBDCs also hinges upon the domestic infrastructure for it and the regulation on international capital movement set by the government(s).

To incorporate the roles of financial intelligence into the model, we assume the following regularity conditions.

Assumption 1. $pH < 1 < S < H$.

Assumption 1 gives rise to a moral hazard problem in banking. Indeed, the bank with limited liability may have an incentive to choose the risky project rather than the safe one in order to lower the expected repayment cost from D to pD per unit of investment. However, such a choice is not in the best interest of the overall economy because $pH < S$. To prevent this moral hazard problem, the depositors can collectively but imperfectly monitor whether the bank makes the "right" project selection. More precisely, if the average value of depositors' types is $\mu_D \in [0, 1]$, the depositors can verify which kind of project the bank runs at no cost with probability μ_D, and the bank can be requested to switch to a different kind of project if necessary. This assumption features the role of depositors' financial intelligence to discipline banks (Flannery and Sorescu, 1996; Martinez Peria and Schmukler, 2001) and thus to alleviate moral hazard in banking: the bank is less likely to succeed in excessive risk-taking as the depositors (in aggregate) are more financially intelligent.

4 Analysis

In this section, we characterize the structure of an equilibrium to identify the effects of the digital currency on the bank's decision on risk-taking.

We first analyze how each type-θ lender optimally makes her lending decision in equilibrium. Let $\mathcal{D}$ denote a subgroup of lenders' types that deposit their money to the bank. Since the cost of monitoring the bank is zero, the depositors always monitor the bank and enforce the bank to switch its project type to a type-G project in the case where the bank's first project choice is B. Thus, $\mathbb{E}[\theta|\theta \in \mathcal{D}]$ is the probability that the bank maintains its initial project selection as type B. Let a deposit contract $D(\leq H)$ be given. When the bank is expected to choose a type-B project, a type-θ lender deposits her wealth to the bank if and only if

$$[(1 - \mathbb{E}[\theta|\theta \in \mathcal{D}])p + \mathbb{E}[\theta|\theta \in \mathcal{D}]]\, D \geq \theta R.$$

Since $pH < 1$ (Assumption 1), the lenders never accepts the bank's offer if $D > S$. Furthermore, when the bank is expected to choose a type-G project, a type-θ lender purchases the digital currency if and only if $D \leq \theta R$.

These observations imply that the lenders' preferences satisfy the single-crossing property, and thus there exists a unique threshold type $\hat{\theta}$ such that a type-θ lender deposits her money to the bank if and only if $\theta \leq \hat{\theta}$ (i.e., $\mathcal{D} = \{\theta \,|\, \theta \leq \hat{\theta}\}$). If the bank chooses a type-B project, it has to switch to type G with probability $\mathbb{E}[\theta|\theta \in \mathcal{D}] = \frac{1}{2}\hat{\theta}$. Hence, $\hat{\theta}$ is the highest value that weakly prefers depositing to the bank, i.e.,

$$\left[\left(1 - \frac{1}{2}\hat{\theta}\right)p + \frac{1}{2}\hat{\theta}\right] D \geq \hat{\theta}R, \tag{1}$$

where the equality holds for $\hat{\theta} < 1$. In contrast, if the bank chooses project type G, a type-θ lender will expect the bank will fully repay D. Hence, $\hat{\theta}$ is the highest value that satisfies

$$D \geq \hat{\theta}R, \tag{2}$$

where the equality holds for $\hat{\theta} < 1$.

Next the bank optimizes its project selection after raising funds from the lenders by offering the deposit contract. We characterize the necessary conditions on $\hat{\theta}$ for which the bank's optimal project selection is type G and B, respectively. Suppose the bank's optimal project choice is type B first. Let D_B denote by the minimum repayment term of the deposit contract that attracts lenders with types $\theta \leq \hat{\theta}$. By (1), D_B is determined by

$$\left[\left(1 - \frac{1}{2}\hat{\theta}\right)p + \frac{1}{2}\hat{\theta}\right] D_B = \hat{\theta} R,$$

which yields

$$D_B(\hat{\theta}) := \frac{\hat{\theta} R}{p + (1-p)\frac{1}{2}\hat{\theta}}. \tag{3}$$

Furthermore, for the repayment term $D_B(\hat{\theta})$, the bank must prefer choosing project B, which is alternatively expressed as

$$p(H - D_B(\hat{\theta})) \geq S - D_B(\hat{\theta}),$$

which yields

$$D_B(\hat{\theta}) \geq \frac{1}{1-p}(S - pH). \tag{4}$$

That is, the bank prefers taking excessive risk only if it expensively borrows funds from the financial consumers. Thus we have the following observation.

Lemma 1. *Define*

$$\underline{\theta}_B := \min\left(\frac{p(S - pH)}{(1-p)\left[R - \frac{1}{2}(S - pH)\right]}, 1\right). \tag{5}$$

If $R \geq \frac{1}{2}(S - pH) > 0$, $\hat{\theta} \geq \underline{\theta}_B$ in the equilibrium where the bank chooses project B. However, if $R \leq \frac{1}{2}(S - pH)$, such an equilibrium cannot exist.

Proof. Fix an equilibrium in which the bank chooses the type-B project. For a given equilibrium threshold type $\hat{\theta}$, plugging $D_B(\hat{\theta})$ in (3) into (4) yields

$$\left[R - \frac{1}{2}(S - pH)\right]\hat{\theta} \geq \frac{p}{1-p}(S - pH). \tag{6}$$

If $R - \frac{1}{2}(S - pH) \leq 0$, the bank strictly dislikes type B. If $R - \frac{1}{2}(S - pH) > 0$, then, by (6) the equilibrium in which the bank chooses project B can exist if $\hat{\theta} \geq \underline{\theta}_B$. *Q.E.D.*

Lemma 1 argues that the bank takes on the excessive risk only when it retains a large group of depositors. Indeed, the bank must offer a very attractive deposit contract to high-type lenders, which will lead to a high repayment cost. To maintain profitability, the bank has an incentive to lower the expected repayment cost by choosing project type B: if the project succeeds, the bank enjoys a higher net return $H - D_B$; if the project fails, the bank need not fulfill its repayment obligation thanks to limited liability.

We next consider an equilibrium in which the bank chooses project G. For a given $\hat{\theta}$, the bank minimizes its repayment cost by offering D_G, which is determined by (2):

$$D_G(\hat{\theta}) = \hat{\theta}R. \tag{7}$$

Furthermore, since it should be optimal for the bank to choose project G, $D_G(\hat{\theta})$ must satisfy the following condition

$$p(H - D_G(\hat{\theta})) \leq S - D_G(\hat{\theta}),$$

or equivalently,

$$D_G(\hat{\theta}) \leq \frac{1}{1-p}(S - pH). \tag{8}$$

In text, the bank dislikes project B only if it gets funds from the depositors at a low deposit rate. (7) and (8) lead to the following lemma.

Lemma 2. *Define* $\overline{\theta}_G$ *as*

$$\overline{\theta}_G := \min\left(\frac{S - pH}{(1-p)R}, 1\right). \tag{9}$$

Then, $\hat{\theta} \leq \overline{\theta}_G$ *in the equilibrium in which the bank chooses project* G*.*

The bank refrains from excessive risk taking when a relatively small group of depositors holds onto their bank accounts rather than adopts the new currency. In this case, the bank does not have to offer generous terms of repayment to attract fairly high-typed depositors. Since the repayment cost is not high, the bank does not have a strong incentive to reduce its liabilities by making a risk-shifting decision. Rather, the bank would maximize the financial return from its investment by choosing the type-G project.

By applying the results of Lemma 1 and 2, we analyze how the bank's optimal project selection varies with R, the lenders' baseline surplus from using the digital currency. To this end, we solve two profit maximization problems for the bank. First, suppose the bank chooses the type-B project. Then the bank's profit function, denoted by $\pi_B(\hat{\theta})$, is

$$\pi_B(\hat{\theta}) := \hat{\theta}\left[\left(1 - \frac{1}{2}\hat{\theta}\right)(pH - D_B) + \frac{1}{2}\hat{\theta}(S - D_B)\right] = \hat{\theta}\left[\left(1 - \frac{1}{2}\hat{\theta}\right)pH + \frac{1}{2}\hat{\theta}S - \hat{\theta}R\right], \tag{10}$$

where the second expression follows from the definition of D_B in (3). For each $R \geq 0$, the bank's maximized profit under its optimal choice of D_B is

$$\overline{\pi}_B(R) := \max_{\hat{\theta} \geq \theta_B} \pi_B(\hat{\theta}) \tag{11}$$

if $R > \frac{1}{2}(S - pH)$. If $R < \frac{1}{2}(S - pH)$, it is not feasible for the bank to choose the type-B project.

Second, the bank's profit function when it chooses project G, denoted by $\pi_G(\hat{\theta})$, is

$$\pi_G(\hat{\theta}) := \hat{\theta}(S - D_G) = \hat{\theta}(S - \hat{\theta}R), \tag{12}$$

where the second expression follows from the definition of D_B in (7). For each R, the bank's profit under its optimal choice of D_G is

$$\overline{\pi}_G(R) := \max_{\hat{\theta} \le \underline{\theta}_G} \pi_G(\hat{\theta}). \tag{13}$$

Then, for each R, the bank's optimal project selection is B if $\overline{\pi}_B(R) \ge \overline{\pi}_G(R)$, and G otherwise. The following theorem reveals that the bank's risk-shifting decision varies with the value of the digital currency, but in a non-monotonic fashion.

Theorem 1. *There exist* $0 \le \underline{R}^* \le \overline{R}^*$ *such that the bank's optimal project selection is* B *if and only if* $R \in (\underline{R}^*, \overline{R}^*]$.

The bank's risk-taking strategy is pinned down by the lenders' baseline surplus from using the digital currency. To attract prospective depositors who have an outside option that originates from the digital currency, the bank needs to offer a more generous terms of repayment than it would absent the digital currency. If R is fairly small, the lenders have an unattractive outside option. Therefore, the bank does not need to offer a very generous deposit contract to raise funds from the lenders, and thus faces relatively low repayment obligations. In this case, to maximize the profit, the bank does not need to take excessive risk. A similar outcome is observed when R is sufficiently high. If the lenders' surplus from the digital currency is sufficiently high, the bank realizes that holding a large share of the lenders, particularly a group of the financially intelligent lenders, is simply unprofitable because the bank must offer a very generous deposit rate. Thus, the bank manages its capital structure by focusing on the group of depositors who can make relatively little use of the new digital currency. Since the bank faces a relatively low overall cost of borrowing in this case, the bank's optimal strategy is to choose project G rather than lowering the borrowing cost by risk-shifting, even though the depositors are on average less likely to perfectly verify the bank's risk-taking behavior.

In contrast, CBDC adversely leads the bank to take on excessive risk when the lenders' surplus from using the new money is neither too low nor too high. A primary reason for the bank's excessive risk taking is the increased borrowing cost due to the intense competition with CBDC. Specifically, financially intelligent lenders can enjoy a relatively large surplus from the digital money. To maintain its large-sized balance sheet, the bank needs to attract these financially intelligent lenders to hold the bank's deposit accounts by offering a highly generous rate. However, pursuing such a business strategy raises the total cost of borrowing, and therefore, exacerbates the bank's capital structure. Moreover, highly financially intelligent lenders choose the digital currency, making the depositors' average ability to monitor the bank's risk-taking behavior weaker compared to the case absent the digital currency. Therefore, to manage its profitability by lowering the repayment cost, the bank strategically passes the investment losses to the depositors by taking on excessive risk.[8]

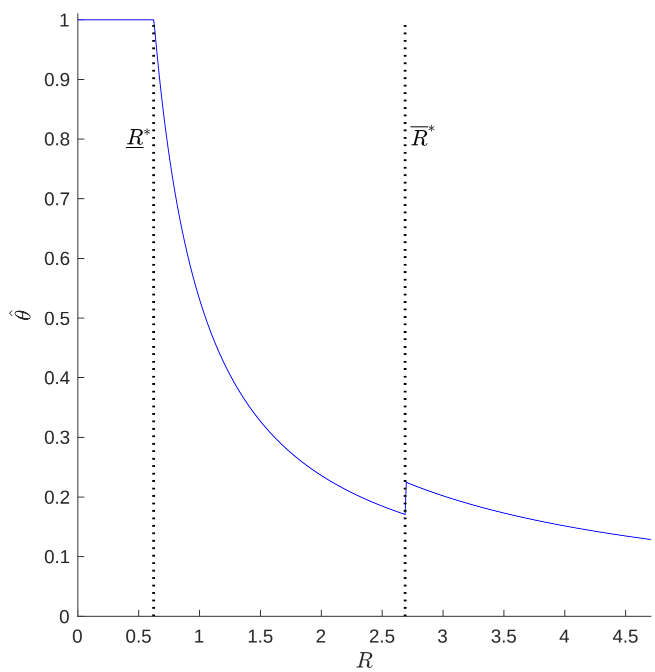

Figure 1 – The threshold type $\hat{\theta}$ ($S = 1.25, H = 2.5$, and $p = 0.34$)

Disintermediation in banking is manifested by the observation that $\hat{\theta}$, representing the size of the bank investment, tends to shrink as R increases. Figure 1 depicts how $\hat{\theta}$ changes with R in a numerical example with $S = 1.25$, $H = 2.5$, and $p = 0.34$.[9] When R is relatively

[8]The main result will remain unaltered even if there are more than one bank that competes for deposits. Rather, competition among banks for deposit taking will increase the equilibrium deposit rate, and therefore, expand the range $(\underline{R}^*, \overline{R}^*]$ (Keeley, 1990).

[9]The numerical results illustrated by Figure 1 generally holds for all parameter values supporting $\underline{R}^* < \overline{R}^*$.

low, there is little change in the bank size compared to the case without CBDC: the bank does not need to offer a generous deposit rate in order to maintain a large-sized deposit pool, and thus has little incentive for excessive risk taking. For intermediate values of $R \in (\underline{R}^*, \overline{R}^*]$, however, the bank downsizes its investment because it is too costly to contain the deposit withdrawal of highly financially intelligent consumers. Interestingly, the bank discreetly scales up its investment as R reaches $\overline{R}^*$: a significant fraction of financial consumers makes deposits back into the bank as the bank is believed to prudently manage its risk. Lastly, as R further exceeds $\overline{R}^*$, the bank once again downsizes its investment by cheaply raising funds from the consumers with relatively low θ's.

With the above result in mind, we further investigate the aggregate surplus from banking when a certain degree of disintermediation happens, i.e., when the introduction of CBDC leads to excessive risk taking by the bank. In this case, we formally define the total surplus from banking as

$$W(R) := \hat{\theta}\left[\frac{1}{2}\hat{\theta}(S-1) + \left(1 - \frac{1}{2}\hat{\theta}\right)(pH-1)\right] \text{ for every } R \in (\underline{R}^*, \overline{R}^*]. \tag{14}$$

Note that the expected return on the bank's investment (per unit capital) strictly worsens from $S-1$ to $\frac{1}{2}\hat{\theta}(S-1) + \left(1 - \frac{1}{2}\hat{\theta}\right)(pH-1)$ when the bank takes excessive risk for intermediate values of R. The following theorem states the overall impact of CBDC on the total surplus from banking.

Theorem 2. *Suppose $\underline{R}^* < \overline{R}^*$. Then there exists a $\breve{R}^* \in (\underline{R}^*, \overline{R}^*]$ such that $W(R)$ is decreasing in R if $R \leq \breve{R}^*$ and $W(R)$ is increasing in R, otherwise.*

When $R \in (\underline{R}^*, \overline{R}^*]$ so that R induces excessive risk taking by the bank, an increase in R creates both negative and positive effects on the aggregate surplus. On the negative side, as discussed above, the bank is more likely to take on excessive risk as R increases: the probability of executing a bad project increases. On the other hand, a larger fraction of financial consumers resorts to CBDC by withdrawing their money from the bank deposits as R increases. That is, a smaller fraction of consumers stays as bank depositors, limiting the total size of "risky" investment the bank makes.

Theorem 2 implies that the effect of the former (latter) is stronger for relatively low (high) R. When R is relatively small, high financially intelligent consumers still stay as bank depositors. Given that the likelihood of the bank's excessive risk taking is low yet, scaling down the bank's investment, resulting from more consumers switching to CBDC, does not substantially improve the efficiency in capital allocation. Therefore, an increase in R yields the net negative impact on the total surplus from banking. In contrast, the likelihood of the bank's excessive risk taking is high when R is relatively large. In this case, an increase in R substantively stems the flow of funds into the bank's risky investment, while it only slightly affects the riskiness of the bank's project given that the bank already takes on highly excessive risk. Hence, an increase in R improves the aggregate surplus from banking.

5 Implications

In order to articulate the implications of Theorem 1 and 2, we need to discuss more detailed background comments on the benefits of using CBDC accounts. Recall that θ, representing the financial intelligence level in our model, captures a lender's capability of taking advantage of the digital currency. In other words, a lender with a high θ is more likely to have a better ability to understand smart contracts, tokenization of real assets, and ideas related to Web 3.0 (e.g., the concept of read/write/own the data). Thus, rather than having indirect investment through a bank deposit, an individual with high θ is more willing to participate in DAOs (Decentralized Autonomous Organizations) to invest in financial products for credit and liquidity creation, yield farming, risk management and so on by using various DeFi (Decentralized Finance) applications. In the real-world financial system, the DeFi-based financial services and products have been mostly undertaken by the established financiers such as investment bankers. Yet, the majority of the public are still reluctant to use them because cryptocurrencies have not been successfully adopted as a means of payment due to their high volatility. As CBDCs, whose values are guaranteed by governmental authorities, are put in place, new financial opportunities will be created in the entire finance sector and thus can be widely available to the general public particularly through their CBDC accounts and smart contracts that will be built on CBDC. However, as previously discussed, public availability of CBDC will vary according to how the infrastructure provision for and the financial regulation of the CBDC usage will be set up. That is, the size of R will be determined by the infrastructure provision and the financial regulation when CBDC is introduced.

With the above aspects in mind, Theorem 1 and 2 have a number of normative implications, each of which depends on how to interpret R in practice. First, our analysis predicts that infrastructure provision for the digital currency influences banks' risk-taking behavior. Financially intelligent customers' preference of holding CBDC to bank deposits will depend on the degree of interoperability between the smart-contract services from CBDC and other financial services provided by the fintech and blockchain industries. In this context, R increases with the ICT infrastructure provision that supports the interoperability of CBDC with existing financial services. For example, if the land and house registry services will be set up to be seamlessly linked to the CBDC ledger (or the central bank's ledger), it will give rise to a better opportunity for individual customers, especially for those with high θ's: they will easily issue and trade tokenized properties by using CBDC without going through banks and other financial intermediaries. Theorem 1 implies that if the infrastructure stays at a fledgling level, private banks will not choose an investment strategy that carries excessively high risk. In this case, the distribution of CBDC will not damage banking stability. However, if the interoperability-related infrastructure develops above a certain limit, the banking stability can be severely weakened due to the deposit withdrawal by highly financially intelligent customers. Another novel feature of our result is that when the sufficiently high level of interoperability is achieved, the CBDC distribution will no longer damage the financial stability, whereas the traditional banking sector will shrink (i.e., the total amount of deposits will decrease).

Second, financial regulations on CBDC can also have a similar impact on the banking stability as does the infrastructure provision. From a regulatory perspective, R tends to be small (high) if the government heavily (lightly) restricts the use of CBDC as a new legal instrument of payments and financial contracts. Our result predicts that obscure financial regulation of CBDC – analogous to intermediate values of R – adversely leads to excessive risk taking by banks. To preserve financial stability in banking, the government would rather either heavily or very lightly regulate the use of CBDC in financial markets, which will hold R either at a sufficiently low level or at a sufficiently high level. It is hard to imagine that the regulation of a country in an early adoption stage will allow individuals to freely use CBDC for various transactions. For example, if a country allows free international transactions through its CBDC, it will increase the possibility of fraud such as money laundering and international online payment scams beyond the country's jurisdiction. However, our theory suggests that the government may need to either maintain the status quo or adopt to a regime with light regulations for the sake of the banking stability in later stages.

Third, Theorem 2 can provide a practical implication in a medium term. We view that a certain degree of disintermediation would be unavoidable after the introduction of CBDC. R can be small at the initial adoption stage. Even if no further favorable ITC infrastructure and regulations are provided by the government, it is expected that R will gradually increase over time thanks to continuous innovations in fintech technology that exploit the advantages of the CBDC usage. Then, it would be a matter of time to observe high risk-taking by banks in a medium term. When such time comes, it is recommended to implement favorable infrastructures and light financial regulations to push R rapidly rather than to maintain/suppress R. The reason is that when the banking instability gets underway, the aggregate surplus from the banking sector increases with R if R becomes relatively large: that is, a large mass of depositors will resort to CBDC for a high R, effectively scaling down the risky investments made by the banks.

Lastly, deposit insurance can be considered as an effective policy solution for the banking instability problem arising from CBDC. Specifically, raising the limit of insured deposits makes CBDC an unattractive choice for financial consumers relative to bank deposits, which will be translated into a lower R in our model. According to Theorem 1 and 2, the deposit insurance limit should be substantially raised to preserve banking stability: banks do not need to offer a high deposit rate to keep the depositors from switching to CBDC, and thus have little incentive to take on excessive risk; highly financially intelligent consumers would rather deposit their money in banks. However, an insufficient increase in the deposit insurance limit can adversely make R lie between $\underline{R}^*$ and $\overline{R}^*$, and therefore, lead to banking instability. Furthermore, policy makers should keep in mind that raising the deposit insurance limit inevitably dampens depositors' incentive to monitor banks' risk-taking, which even possibly outweighs the aforementioned positive effects (Demirgüç-Kunt and Detragiache, 2002). To address this problem, financial regulatory agencies, on behalf of the individual depositors, are required to collectively monitor the banks' risk-taking behavior more closely.

6 Conclusion

We have investigated the impact of CBDC on banking stability associated with the programmability of CBDC. Specifically, we show that when customers have heterogeneous financial intelligence, banking stability is exacerbated if the baseline surplus from using CBDCs has intermediate values. This result provides several normative implications that are novel relative to the existing literature, which mostly focused on the issues derived from the interest-bearing feature of the CBDC or from the superiority of CBDC relative to physical cash and treasury bills. We also found that the aggregate surplus from banking is non-monotonic with R, which provides novel insight into a potential government policy when the financial instability from the introduction of CBDC is put in place. We hope our research sheds light on the debate over how to provide the appropriate infrastructure arrangement and financial regulations over time (from early adoption to full-fledged development stages).

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Appendix

A Proofs

Proof of Theorem 1:

Proof. We first present several observations used in the proof.

Claim 1. *$\pi_B(\hat{\theta})$ is maximized at $\hat{\theta}^*_B := \frac{pH}{2\left[R-\frac{1}{2}(S-pH)\right]}$.*

Proof. From (10), we know $\pi_B(\hat{\theta})$ is strictly concave in $\hat{\theta}$. Thus, by the first order condition, it is straightforward that $\pi_B(\hat{\theta})$ is maximized at a unique extreme point $\hat{\theta}^*_B = \frac{pH}{2\left[R-\frac{1}{2}(S-pH)\right]}$. *Q.E.D.*

Claim 2. *$\pi_G(\hat{\theta})$ is maximized at $\hat{\theta}^*_G := \frac{S}{2R}$.*

Proof. From (12), we know $\pi_G(\hat{\theta})$ is strictly concave in $\hat{\theta}$. Thus, by the first order condition, it is straightforward that $\pi_G(\hat{\theta})$ is maximized at a unique extreme point $\hat{\theta}^*_G = \frac{S}{2R}$. *Q.E.D.*

Claim 3. *$\underline{\theta}_B \leq 1$ if and only if $R \geq \frac{1}{2}\left(\frac{1+p}{1-p}\right)(S-pH)$.*

Proof. By definition of $\underline{\theta}_B$ in (5), we have

$$\underline{\theta}_B \leq 1 \iff \frac{p(S-pH)}{(1-p)\left[R-\frac{1}{2}(S-pH)\right]} \leq 1 \iff R \geq \frac{1}{2}\left(\frac{1+p}{1-p}\right)(S-pH).$$

Q.E.D.

Claim 4. *$\overline{\theta}_G \leq 1$ if and only if $R \geq \frac{1}{1-p}(S-pH)$.*

Proof. By definition of $\overline{\theta}_G$ in (9), we have

$$\overline{\theta}_G \leq 1 \iff \frac{(S-pH)}{(1-p)R} \leq 1 \iff R \geq \frac{1}{1-p}(S-pH).$$

Q.E.D.

Claim 5. *Both* $\hat{\theta}_B^* < 1$ *and* $\hat{\theta}_G^* < 1$ *if and only if* $R > \frac{1}{2}S$.

Proof. First, we have

$$\hat{\theta}_B^* < 1 \iff \frac{pH}{2\left[R - \frac{1}{2}(S - pH)\right]} < 1 \iff R > \frac{1}{2}S.$$

Second, we have

$$\hat{\theta}_G^* < 1 \iff \frac{S}{2R} < 1 \iff R > \frac{1}{2}S.$$

Q.E.D.

Claim 6. *If* $\underline{\theta}_B \gtrless \hat{\theta}_B^*$ *for some* R*, then* $\underline{\theta}_B \gtrless \hat{\theta}_B^*$ *for every* $R' \neq R$ *such that* $\min\{\underline{\theta}_B, \hat{\theta}_B^*\} < 1$. *Similarly, if* $\hat{\theta}_G^* \gtrless \underline{\theta}_G$ *for some* R*, then* $\hat{\theta}_G^* \gtrless \underline{\theta}_G$ *for every* $R' \neq R$ *such that* $\min\{\underline{\theta}_G, \hat{\theta}_G^*\} < 1$.

Proof. Suppose $\underline{\theta}_B \gtrless \hat{\theta}_B^*$ for some R. Then we must have

$$\frac{p(S - pH)}{(1-p)\left[R - \frac{1}{2}(S - pH)\right]} \gtrless \frac{pH}{2\left[R - \frac{1}{2}(S - pH)\right]} \iff \frac{S - pH}{(1-p)H} \gtrless \frac{1}{2}.$$

The last expression implies that R does not determine whether or not $\underline{\theta}_B \gtrless \hat{\theta}_B^*$. Hence, we have $\underline{\theta}_B \gtrless \hat{\theta}_B^*$ for every $R' \neq R$ such that either $\hat{\theta}_B^* \neq 1$ or $\underline{\theta}_B \neq 1$.

Next, suppose $\hat{\theta}_G^* \gtrless \underline{\theta}_G$ for some R. Then we must have

$$\frac{S}{2R} \gtrless \frac{(S - pH)}{(1-p)R} \iff \frac{1}{2} \gtrless \frac{S - pH}{(1-p)S}.$$

Again the last expression implies that R does not determine whether or not $\underline{\theta}_G \gtrless \hat{\theta}_G^*$. Hence, we have $\hat{\theta}_G^* \gtrless \underline{\theta}_G$ for every $R' \neq R$ such that either $\hat{\theta}_G^* \neq 1$ or $\underline{\theta}_G \neq 1$. *Q.E.D.*

If $R \leq \frac{1}{2}(S - pH)$, project type B is not a feasible choice, and therefore, the bank must choose project G. Hence, we throughout restrict our attention to the cases $R > \frac{1}{2}(S - pH)$.

If $R > \frac{1}{2}(S - pH)$, there are three possible cases, *(i)* $\frac{1}{2}S < \frac{1}{2}\left(\frac{1+p}{1-p}\right)(S - pH)$; *(ii)* $\frac{1}{2}S \in \left[\frac{1}{2}\left(\frac{1+p}{1-p}\right)(S - pH), \frac{1}{1-p}(S - pH)\right)$; *(iii)* $\frac{1}{2}S \geq \frac{1}{1-p}(S - pH)$. We will characterize the bank's optimal project selection rule with respect to R for each case.

Case (i): Suppose $\frac{1}{2}S < \frac{1}{2}\left(\frac{1+p}{1-p}\right)(S-pH)$. For every $R \leq \frac{1}{2}S$, we have $\underline{\theta}_B = \overline{\theta}_G = 1 \leq \min\{\hat{\theta}_B^*, \hat{\theta}_G^*\}$. Thus, the bank gets $\pi_B(1)$ if it chooses project B and $\pi_G(1)$ if it chooses project G, respectively. Furthermore, since $S - pH > 0$, we have $\pi_G(1) > \pi_B(1)$, which implies that it is optimal for the bank to choose project G.

Next consider $\frac{1}{2}S < R$. Since $\frac{1}{2}S < \frac{1}{2}\left(\frac{1+p}{1-p}\right)(S-pH) < \frac{1}{1-p}(S-pH)$, it follows from Claims 3 - 6 that $\hat{\theta}_B^* < \underline{\theta}_B = 1$ when $R = \frac{1}{2}\left(\frac{1+p}{1-p}\right)(S-pH)$. Thus, we have $\hat{\theta}_B^* < \underline{\theta}_B$ for every $R > \frac{1}{2}S$. Similarly, since $\hat{\theta}_G^* < \overline{\theta}_G = 1$ when $R = \frac{1}{1-p}(S-pH)$, we have that $\hat{\theta}_G^* < \overline{\theta}_G$ for every $R > \frac{1}{2}S$. In sum, $\hat{\theta}_B^* < \underline{\theta}_B$ and $\hat{\theta}_G^* < \overline{\theta}_G$ for every $R > \frac{1}{2}S$. This observation implies

$$\max_{\hat{\theta} \geq \underline{\theta}_B} \pi_B(\hat{\theta}) = \pi_B(\underline{\theta}_B) \quad \text{and} \quad \max_{\hat{\theta} \leq \underline{\theta}_G} \pi_G(\hat{\theta}) = \pi_G(\hat{\theta}_G^*) \quad \text{for every } R > \frac{1}{2}S.$$

If $\underline{\theta}_B = 1$ (i.e., $R \leq \frac{1}{2}\left(\frac{1+p}{1-p}\right)(S-pH)$ by Claim 3), we have that $\pi_B(\underline{\theta}_B) < \pi_G(1) \leq \pi_G(\hat{\theta}_G^*)$. Thus it is optimal for the bank to choose project G for every $R \leq \frac{1}{2}\left(\frac{1+p}{1-p}\right)(S-pH)$. If $\underline{\theta}_B < 1$ (i.e., $R > \frac{1}{2}\left(\frac{1+p}{1-p}\right)(S-pH)$), we have $\pi_B(\underline{\theta}_B) = \frac{p}{1-p}(H-S)\underline{\theta}_B$ and $\pi_G(\hat{\theta}_G^*) = \frac{S^2}{2R}$. Then we have

$$\pi_B(\underline{\theta}_B) \geq \pi_G(\hat{\theta}_G^*) \iff 2\left[\frac{p}{(1-p)S}\right]^2 (H-S)(S-pH) \geq 1 - \frac{S-pH}{2R} := f(R).$$

Since $f(R)$ is zero when $R = \frac{1}{2}(S-pH)$ and is strictly increasing in R, there exists $\overline{R}^* \geq \frac{1}{2}\left(\frac{1+p}{1-p}\right)(S-pH)$ such that the bank's optimal project selection is B if and only if $R \in \left(\frac{1}{2}\left(\frac{1+p}{1-p}\right)(S-pH), \overline{R}^*\right]$. By letting $\underline{R}^* := \frac{1}{2}\left(\frac{1+p}{1-p}\right)(S-pH)$, we have the desired result.

Case (ii): Suppose $\frac{1}{2}S \in \left[\frac{1}{2}\left(\frac{1+p}{1-p}\right)(S-pH), \frac{1}{1-p}(S-pH)\right)$. Let us first consider the case in which $R \leq \frac{1}{2}\left(\frac{1+p}{1-p}\right)(S-pH)$, for which we have $\underline{\theta}_B = \overline{\theta}_G = 1 \leq \min\{\hat{\theta}_B^*, \hat{\theta}_G^*\}$ (recall Claims 3 and 4). Since $\hat{\theta}_B^* \geq 1$ and $\hat{\theta}_G^* \geq 1$ by Claim 5, the bank gets $\pi_B(1)$ if it chooses project B and $\pi_G(1)$ if it chooses project G, respectively. Since $\pi_G(1) > \pi_B(1)$, the bank's optimal choice is project G.

Next consider the case in which $R \in \left(\frac{1}{2}\left(\frac{1+p}{1-p}\right)(S-pH), \frac{1}{2}S\right]$, where we have $\underline{\theta}_B < 1 \leq \hat{\theta}_B^*$ and $\hat{\theta}_G^* \geq \overline{\theta}_G = 1$. Since $\hat{\theta}_B^* \geq 1$ and $\hat{\theta}_G^* \geq 1$, the bank gets $\pi_B(1)$ if it chooses project B

and $\pi_G(1)$ if it chooses project G, respectively. Hence, the bank's optimal choice is project G.

Lastly consider the case $R > \frac{1}{2}S$. Note from Claim 6 that we have $\underline{\theta}_B < \hat{\theta}^*_B$ and $\hat{\theta}^*_G < \overline{\theta}_G$ for every $R > \frac{1}{2}\left(\frac{1+p}{1-p}\right)(S - pH)$. Furthermore, by Claim 1 and 2, the bank gets $\pi_B(\hat{\theta}^*_B)$ if it chooses project B and $\pi_G(\hat{\theta}^*_G)$ if it chooses project G, respectively. Then, we have

$$\pi_B(\hat{\theta}^*_B) < \pi_G(\hat{\theta}^*_G) \iff \left(\frac{pH}{S}\right)^2 < 1 - \frac{S - pH}{2R} = f(R).$$

Since $f(R)$ is strictly increasing in R, $f(R) = \frac{pH}{S}$ at $R = \frac{1}{2}$, and $pH < S$, the above inequality holds for all $R > \frac{1}{2}S$, which means that the bank chooses project G in this case.

Case (iii): Suppose $\frac{1}{2}S > \frac{1}{1-p}(S - pH)$. Let us first consider the case of $R \leq \frac{1}{2}S$, where we have $\hat{\theta}^*_B \geq 1 \geq \underline{\theta}_B$ and $\hat{\theta}^*_G = 1 \geq \overline{\theta}_G$. Hence, the bank gets $\pi_B(1)$ if it chooses project B and $\pi_G(1)$ if it chooses project G, respectively. Since $\pi_B(1) < \pi_G(1)$, the bank's optimal choice is project G.

Next consider the case of $R > \frac{1}{2}S$. Then, by Claim 6, we have $\underline{\theta}_B < \hat{\theta}^*_B < 1$ since $\frac{1}{2}S > \frac{1}{2}\left(\frac{1+p}{1-p}\right)(S - pH)$ and $\overline{\theta}_G < \hat{\theta}^*_G < 1$ since $\frac{1}{2}S > \frac{1}{1-p}(S - pH)$. Therefore, the bank gets $\pi_B(\hat{\theta}^*_B)$ if it chooses project B and $\pi_G(\overline{\theta}_G)$ otherwise. Furthermore, we have

$$\pi_B(\hat{\theta}^*_B) \geq \pi_G(\overline{\theta}_G) \iff \frac{[(1-p)pH]^2}{2p(H-S)(S-pH)} \geq 1 - \frac{S - pH}{2R} = f(R). \tag{15}$$

Note that $f(R)$ is zero when $R = \frac{1}{2}(S - pH)$ and is strictly increasing in R. Suppose there exists a $\overline{R}^* \geq \frac{1}{2}S$ at which the inequality in (15) becomes equal. Then, the bank's optimal choice is project B if and only if $R \in \left(\frac{1}{2}S, \overline{R}^*\right]$. By letting $\underline{R}^* := \frac{1}{2}S$, we have the desired result. If no such $\overline{R}^*$ exists, then the bank's optimal project selection is G. *Q.E.D.*

Proof of Theorem 2:

Proof. First recall from the proof of Theorem 1 that $\hat{\theta} = \hat{\theta}^*_B$ for all $R \in (\underline{R}^*, \overline{R}^*]$, which implies $\hat{\theta}$ is continuous and strictly decreasing in R for all $R \in (\underline{R}^*, \overline{R}^*]$.

From (2), we have $\frac{d}{d\hat{\theta}}W = (pH - 1) + \hat{\theta}(S - pH)$. Since $pH < 1$ and $S > pH$ by Assumption 1, we have that $\frac{d}{d\hat{\theta}}W > 0$ if and only if $\hat{\theta} > \frac{1-pH}{S-pH}$. Since $\frac{1-pH}{S-pH} < 1$ and $\hat{\theta}$

is continuous and strictly decreasing in R, there exists a $\breve{R}^* \in (\underline{R}^*, \overline{R}^*]$ such that $W(R)$ is strictly decreasing in R if and only if $R < \breve{R}^*$, which is the desired result. *Q.E.D.*