Discrete Analysis Seminar

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Towards a high-dimensional Dirac's theorem

Dirac's theorem determines the sharp minimum degree threshold for graphs to contain perfect matchings and Hamiltonian cycles. There have been various attempts to generalize this theorem to hypergraphs with larger uniformity by considering hypergraph matchings and Hamiltonian cycles. In this paper, we consider another natural generalization of the perfect matchings, Steiner triple systems. As a Steiner triple system can be viewed as a partition of pairs of vertices, it is a natural high-dimensional analogue of a perfect matching in graphs.

We prove that for sufficiently large integer n with $n \equiv 1$ or $3 \pmod 6$, any n-vertex 3-uniform hypergraph H with minimum codegree at least $\left(\frac{3+\sqrt{57}}{12} + o(1)\right) n = (0.879... + o(1))n$ contains a Steiner triple system. In fact, we prove a stronger statement by considering transversal Steiner triple systems in a collection of hypergraphs. We conjecture that the number $\frac{3+\sqrt{57}}{12}$ can be replaced with $\frac{3}{4}$ which would provide an asymptotically tight high-dimensional generalization of Dirac's theorem.

Date: 20th November, 2023

Time: 5:00pm – 6:00pm

Location: 254, Science Building

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