Seminar 2023

Math

The local solubility for homogeneous polynomials with random coefficients over thin sets

연기석

Purdue University

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ABSTRACT. Let d and n be natural numbers greater or equal to 2. Let $\langle \boldsymbol{a}, \nu_{d,n}(\boldsymbol{x}) \rangle \in \mathbb{Z}[\boldsymbol{x}]$ be a homogeneous polynomial in n variables of degree d with integer coefficients $\boldsymbol{a}$, where $\langle \cdot, \cdot \rangle$ denotes the inner product, and $\nu_{d,n} : \mathbb{R}^n \to \mathbb{R}^N$ denotes the Veronese embedding with $N = \binom{n+d-1}{d}$. Consider a variety $V_{\boldsymbol{a}}$ in $\mathbb{P}^{n-1}$, defined by $\langle \boldsymbol{a}, \nu_{d,n}(\boldsymbol{x}) \rangle = 0$. In this paper, we examine a set of these varieties defined by

$$\mathbb{V}_{d,n}^{P}(A) = \{V_{\boldsymbol{a}} \subset \mathbb{P}^{n-1} \mid P(\boldsymbol{a}) = 0,\ \|\boldsymbol{a}\|_\infty \leq A\},$$

where $P \in \mathbb{Z}[\boldsymbol{x}]$ is a non-singular form in N variables of degree k with $2 \leq k \leq C(n,d)$ for some constant $C(n,d)$ depending at most on n and d. Suppose that $P(\boldsymbol{a}) = 0$ has a nontrivial integer solution. We confirm that the proportion of varieties $V_{\boldsymbol{a}}$ in $\mathbb{V}_{d,n}^{P}(A)$, which are everywhere locally soluble, converges to a constant c_P as $A \to \infty$. In particular, if there exists $\boldsymbol{b} \in \mathbb{Z}^N$ such that $P(\boldsymbol{b}) = 0$ and the variety $V_{\boldsymbol{b}}$ in $\mathbb{P}^{n-1}$ admits a smooth $\mathbb{Q}$-rational point, the constant c_P is positive.

연세대학교 수학계산학부

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