Tying in Two-Sided Markets with Below-Cost or Negative Pricing

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Abstract

We show that tying can be profitable in two-sided markets even if below-cost or negative pricing is allowed. With the coexistence of two consumer groups (one treating tying and tied goods as perfect complements and the other as independent products), a tying-good monopolist may face difficulties in extracting rent and wish to use tying to directly capture the large advertising revenue created in the complementary segment. Such tying normally reduces consumer surplus and total welfare. Our theory of tying can be applied to the practice of self-preferencing or requiring pre-installation as in the Google Android and Shopping cases.

1   Introduction

This study examines monopoly incentives for tying and its welfare effects in two-sided markets in which firms can charge a price below cost or even a negative price on the consumer side. Although tying is still prevalent in ordinary one-sided markets, more attention has recently been paid to the various tying practices in two-sided markets. Notable is the antitrust case against Google in which the European Commission found

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Google's practice of tying its app store (the Play Store) and other apps such as Google Search and Chrome browser illegal. Other examples include credit cards and department stores/duty-free shops tied in the form of bundled discounts and online portals and search engines favoring their own shopping services or mobile payment systems when consumers use their platform.

We consider the situation in which a tying-good monopolist competes against a more efficient rival in the tied-good market. Both tying- and tied-good markets are two-sided.[1] Assume that the fixed costs of production or entry, if any, are sunk for all the firms in the markets. Thus, competitive pressure is always present in the tied-good market and entry deterrence or exit inducement by committing to tying as in Whinston (1990) is not an issue. As shown by Choi and Jeon (forthcoming), the Chicago critique of tie-ins fails to hold, and tying can be profitable in two-sided markets if negative prices are not permitted and marginal costs are low. In the independent-products case, the monopolist can use tying to circumvent the non-negative price constraint in the tied-good market without inviting aggressive responses by the rival firm. In the complementary-products case, the Chicago-style price squeeze ceases to operate under the binding non-negative price constraint and therefore the monopolist may wish to use tying to extract the extra surplus created in the tied-good market. If firms can freely charge negative prices or marginal costs are high, however, the tying motivation disappears, as the standard Chicago logic again takes effect with unconstrained price competition.

Optimal pricing in two-sided markets typically involves below-cost pricing on one side of the market, with the entailing loss recouped on the other side (see Armstrong (2006) and Rochet and Tirole (2006)). When the marginal cost is close to zero, as in markets for information goods, below-cost pricing means negative prices. In fact, finding examples of negative or pseudo-negative prices is not difficult. Cashback or other non-monetary rewards are usually provided for the use of a credit card at certain places of purchase.[2] Such reward programs are free from moral hazard or opportunistic behavior by consumers since compensation is directly linked to usage, which generates extra revenue on the other side of the market.[3] Free e-mail accounts and cloud storage offered to subscribers of portals

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[1]Two-sidedness for the tying good is not essential to derive our main results, as shown later.

[2]Cashback is an incentive program operated by credit card companies under which a percentage of the amount spent is paid back to the cardholder.

[3]Without one-to-one linkage between purchases and extra revenue, negative pricing would suffer from moral hazard and adverse selection, as consumers simply demand the good to make money but never use it. See Choi and Jeon (forthcoming) for more discussion on this issue.

and SNS platforms can be considered to be a pseudo-negative price. Although there is no monitoring mechanism, those programs are not seriously vulnerable to opportunistic behavior since rewards are non-monetary and resale is not possible. More vivid examples of negative pricing include reward apps that provide points that can be traded for gift cards at partner retailers (Swagbucks, Shopkick). Cashback rebates (Ibotta) or actual hard cash (Cashslide, UserTesting) are provided in return for purchasing using the apps, searching the web, answering surveys, or offering feedback on a website. Those apps are usually financed through sponsorships with the brands they partner with or by selling personal data to advertisers or other third parties.[4] Providing free amenities and cultural programs by department stores or shopping malls is an example of below-cost pricing for goods and services with relatively high marginal or fixed costs. With the non-monetary nature and restrictions on resale, these are also less open to the risk of moral hazard in consumption. More examples of freebees with high marginal costs can be found in Evans and Schmalensee (2016).

Given these observations, the question arises why tying occurs in two-sided markets when firms can charge *below-cost or negative prices.* One answer lies in the coexistence of two groups of consumers, one treating tying and tied goods as perfect complements and the other as independent products. It is not uncommon in economic analyses to divide consumers into separate groups according to consumption patterns. To the best of our knowledge, however, no study has thus far analyzed tying incentives in such an environment. Furthermore, we allow the size of the extra rent generated on the other side of the tied-good market to differ between the two groups. For example, consumers of online applications can be divided into two groups. Some people use mobile platforms (tying goods) and applications (tied goods) in tightly coupled forms, like complements. Others use platforms and applications rather independently of each other, like people who use a mobile phone mainly for communication and access applications such as search engines and Internet browsers mostly on a desktop. It seems plausible to assume that complementary users are more likely to be exposed and more responsive to online advertising than independent users. Google's internal data show that more searches take place on mobile devices than on computers in 10 countries including the United States and Japan, which

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[4]For more detail about how these reward apps operate, see the New York Times article "4 Free Apps That Can Earn You Extra Cash" by Kristin Wong (https://www.nytimes.com/2018/02/13/smarter-living/cash-back-apps.html).

indicates that mobile users tend to generate more advertising revenue than desktop users.[5] In addition, the results of a study carried out by comScore, Inc. using survey-based data to measure attitudinal changes among consumers exposed to mobile advertisements reveal that mobile ads are more effective than desktop ads in gaining consumer awareness, favorability, likelihood to recommend, and purchase intent (see Fulgoni (2015)). In particular, mobile advertising lifts consumers' purchase intent nearly four times more than desktop advertising.[6] Similarly, premium credit cards and department stores are more like complements for high-income consumers who shop frequently. On the contrary, those two are rather independent goods for low-income consumers who occasionally visit department stores and use ordinary cards or cash. The former users typically spend more and buy expensive products and, therefore, are more valuable to department stores, which can then collect high slotting fees from merchants. Then, the Chicago logic fails to hold and the monopoly incentive for tying can reappear if the extra revenue generated in the complementary group is sufficiently large. We uncover two distinct mechanisms by which tying raises monopoly profits in two-sided markets when firms can freely charge below-cost or negative prices.

First, consider the case in which the relative size of the complementary group is sufficiently large for the monopolist to wish to sell its tying good or the bundle only to the consumers in that group. According to the Chicago logic in the case of perfect complements, the monopolist prefers separate sales to tying because it wants to extract the surplus the efficient rival creates using a price squeeze. In the present setup, however, the monopolist's ability to squeeze prices is constrained by the break-even constraint of the rival firm if the additional revenue generated on the other side of the tied-good market is larger in the complementary group than in the independent group and the third-degree price discrimination is not feasible in the tied-good market. Facing this imperfect rent extraction under separate sales, the monopolist may then wish to tie its products to directly capture the large additional revenue generated in the complementary group.

Next, consider the case in which the relative size of the independent group is sufficiently large so that the monopolist chooses to sell its tying good or the bundle to the consumers in both groups. In this full participation case, there is no price squeeze for rent extraction and the monopolist sells the tying good at the monopoly price under separate sales. So, in this case it does not matter whether price discrimination is possible or not for the tied

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[5]See "Building for the next moment" on May 5, 2015 in the Google Ads blog.

[6]See Figure 3 in Fulgoni (2015) for more detail.

good. Under tying, however, the monopolist faces price competition against the efficient rival. The Chicago school critique says that tying only reduces the monopoly profit in this case, since it has to lower the price of the bundle to compensate consumers for using its inferior tied good. In our model, however, the additional revenue generated from the complementary group on the other side of the tied-good market can be partly insulated from this competitive pressure since the price competition between the bundle and the rival's tied good is only effective in the independent group. Therefore, the monopolist may opt for tying if the additional revenue extracted from the complementary consumers is sufficiently large to outweigh the profit loss from selling the bundle at a lower price.

When the two groups are more or less similar in size, demand for the tying good may either contract or expand with tying. A similar logic can be applied in these cases. Tying can be profitable if it allows the monopolist to extract the large extra revenue generated from the complementary group on the other side of the tied-good market. The coexistence of the two groups is *necessary* for tying to be profitable: both mechanisms fail to operate if the complementary or independent group exists alone.

Profitable tying usually reduces consumer surplus and social welfare because it induces some consumers to purchase the bundle containing the inferior or less preferred tied good; in addition, price competition is less intense under tying than separate sales. However, we cannot exclude the possibility that consumer surplus and social welfare increase under tying, although the condition for this to happen is stringent. First, the sales volume of the tying good should increase under tying, which is similar to the usual necessary condition for third-degree price discrimination to increase social welfare.[7] Then, total welfare may increase under tying if the gain from the demand expansion is greater than the loss from the exclusion of efficient rivals in the tied-good market. Furthermore, when the monopolist sells the bundle to the consumers in both groups, it lowers the bundle price to win the price competition against the efficient rival. Consumers may benefit from tying in this situation if the bundle price is sufficiently reduced.

The possibility of multi-homing intensifies the price competition between the bundle and the rival's standalone product and thus makes tying less profitable. Nonetheless, the tying motivation still exists if the proportion of multi-homers is not that large and the extra revenue captured in the complementary group is sufficiently large. Not surprisingly, tying is more likely to increase consumer surplus with multi-homing. The baseline model

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[7]However, with horizontal differentiation for the tied good, tying may increase consumer surplus even without expanding demand, as shown later.

is extended to incorporate horizontal differentiation in the tied-good market and similar results are obtained. Tying here creates another inefficiency by having some consumers use products they do not like. Again, tying can be profitable if the extra revenue generated in the complementary group is sufficiently large. We show that our model can be used to analyze the economic impact of real-world antitrust events such as the Google Android case. We also offer an alternative modeling of the coexistence of complementary and independent groups, where consumers have different expectations of the complementarity of the tying and tied goods. Finally, the qualitative results of our analysis are unchanged even if there exist multiple subgroups generating different extra revenue in each of the complementary and independent groups. The model setting would be essentially the same if we redefine the average advertising profit for each group.

Literature Review There is a substantial body of literature on the theory of tying. The most closely related study to our analysis is Choi and Jeon (forthcoming). Our first tying mechanism is somewhat similar to that identified for the complementary-products case in Choi and Jeon's model. The monopolist faces difficulty in extracting rent under separate sales in both models. However, the source of the imperfect rent extraction is different between the two models: the non-negative price constraint in Choi and Jeon compared with the rival's break-even constraint under the coexistence of the complementary and independent groups in our model. Further, our first tying mechanism shares the same spirit as Carlton and Waldman (2012), who show that a tying-good monopolist may wish to tie its products to capture the surplus consumers obtain from future upgrades of tied goods. The extraction of future rent is not possible in separate sales if consumers are not willing to pay in advance for future utilities and the monopolist cannot set the price of its tying good contingent on the future availability of upgrades. Once again, the tying motive in their model, that is, the inability to extract the rent from future upgrades, is different than ours.

On the contrary, our second tying mechanism is related to the one Choi and Jeon found for the case of independent products. In Choi and Jeon, tying can be profitable because the extra revenue from the tied good is not dissipated since the rival cannot price aggressively under non-negative price constraints. As noted before, this mechanism does not work if firms can freely charge below-cost or negative prices: the rent will be fully dissipated under tying with unconstrained price competition.[8] Our analysis reveals

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[8]Choi and Jeon (forthcoming) also show that when intergroup network effects are strong in the tied-good market and consumers have heterogeneous preferences for the tying good, the monopolist can

that another mechanism can prevent the full dissipation of the rent under tying—even if below-cost or negative pricing is feasible. In our model, the extra revenue generated from the complementary consumers for the tied good, if it is large, is partly immune to price competition under tying, which pertains only to the independent group. The monopolist can thus extract the saved rent using tying, and tying becomes profitable if the extra rent obtained from the complementary group is larger than the loss from price competition.

Consumers benefit from tying because of price cuts in Choi and Jeon (forthcoming).[9] On the contrary, in our model, tying mostly reduces consumer surplus by making competition less intense, although the opposite case can happen when demand expands with tying or multi-homing is possible on the consumer side. The rival firm's profit tends to decrease under tying in Choi and Jeon since price competition is more intense in both the complementary and the independent-products cases. On the contrary, in our model, the tied-good rival's profit increases under tying whenever the monopolist chooses to sell the bundle only to the consumers in the complementary group. In this case of partial participation, tying leads to a collusive outcome, where the monopolist sells the bundle to the complementary group and the rival firm sells the tied good to the independent group, each at the monopoly price. Such collusive pricing is feasible in our model because the two groups of consumers coexist in the market. This result is reminiscent of the findings of Carbajo, Meza, and Seidman (1990) and Chen (1997), who show that bundling relaxes price competition by segmenting the market. Both studies address a one-sided market and their market-segmentation results critically depend on the heterogeneity of consumer preferences for the goods. On the contrary, our analysis demonstrates that a similar competition-dampening effect can occur in two-sided markets composed of homogeneous consumers provided complementary and independent groups coexist.

Other studies have examined tying incentives in two-sided markets in different contexts. Choi (2010) shows that tying in two-sided markets can be welfare-improving when multi-homing is possible. Amelio and Jullien (2012) show that tying can be used as a tool to introduce implicit subsidies to the targeted side of a two-sided market under non-negative price constraints. Tying here can benefit consumers by raising participation in

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use tying as a strategic device to induce consumers' coordination around the bundled product. This tying incentive, although independent of the non-negative price constraint, requires a high degree of two-sidedness for the *tying* good. By contrast, the tying incentive in our analysis is independent of the two-sided nature of the tying-good market.

[9]Consumer surplus may increase under tying in their alternative model with intergroup network effects in the tied-good market.

both sides in the case of a monopoly platform.[10] They provide efficiency rationales for tying in two-sided markets. The mechanisms for tying to increase monopoly profits in these models are starkly different from ours.

Motivated by the recent Google Android case, Etro and Caffarra (2017) present a theory of anticompetitive tying in two-sided markets. They show that a monopolist may wish to tie to extract the rent coming from the superior quality of its tying good when it had to commit to a zero price for the tying good. Here, an imperfect rent extraction occurs in the market for the tying good, whereas in ours and Choi and Jeon's model, the target of the rent extraction is the extra revenue created in the tied-good market. Further, de Cornière and Taylor (2018) analyze bundling incentives in a vertical market in which a downstream firm procures components from upstream suppliers and positive wholesale markups are induced by contractual frictions. They show that a tying-good monopolist upstream has an incentive to bundle its products to reduce the tied-good rival's willingness to offer slotting fees to the downstream firm, thereby capturing more of the industry profit. However, their model does not explicitly consider the two-sidedness of the market.

More traditional studies of leverage theory on tying examine the exclusion of efficient rivals in the presence of economies of scale.[11] For example, in Whinston (1990), tying is profitable only if the monopolization of the tied-good market follows via entry deterrence or exit inducement and the successful exclusion of rivals requires a commitment to tying. In a similar vein, Choi and Stefanadis (2001) and Carlton and Waldman (2002) provide dynamic leverage theories of tying in the case of complementary products. They show that tying can be used to protect the monopolized market from new entry rather than monopolize the tied-good market. In our model, the exclusion of rivals is not an issue, although it can happen as a result of tying. Further, our analysis focuses on the role of tying in extracting the additional surplus created on the other side of the tied-good market rather than monopolizing the tied-good market itself. Thus, the intuition and results obtained in our analysis can be applied to the practice of raising rivals' costs that do not involve exclusion of rivals, such as self-preferencing and requiring pre-installation as a default. In our setting, the tying-good monopolist has incentives for tying as long as

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[10]With platform competition, however, the welfare effect is ambiguous and depends on the degree of asymmetric externalities on the two sides.

[11]Other stands of the literature explore the efficiency and price discrimination motives of tying. Fumagalli, Motta, and Calcagno (2018) provide an excellent survey of these works.

it increases its own market share in the complementary segment of the tied-good market. In this sense, our analysis is also related to Nalebuff (2004) who explores tying incentives without commitment when the tying firm acts as a Stackelberg leader in the price game. His model, however, deals with one-sided markets and differs from ours in the mechanism by which tying increases profits.

Our analysis is also complementary to the growing literature on platform envelopment strategies, whereby a dominant platform operating in a two-sided market enters another platform market by leveraging its shared user relationships in the two markets. See the seminal paper by Eisenmann et al. (2011) who explain how one platform provider can enter in markets subject to network effects and switching costs by bundling its own functionality with that of the target's so as to leverage shared user relationships and common components. Our results provide a theoretical background for the economic motivation of platform envelopment, where the main source of revenue for the envelopment strategy is advertising profits generated in the other side of the target market.[12]

Overall, our work complements the existing literatures by providing new rationales for tying and its welfare implications in two-sided markets in which two groups of consumers with complementary and independent demand coexist and firms can charge below-cost or negative prices. Parameterizing the relative size of the two groups and the degree of two-sidedness, we analyze monopoly tying incentives in a unified framework encompassing the complementary-products and independent-products models as special cases.

2   A baseline model of homogeneous consumers

Consider two-sided markets for products A and B. Consumers, whose total mass is normalized to 1, are exogenously divided into two groups. In group 1, there is a mass $\lambda \in [0, 1]$ of identical consumers who treat products A and B as perfect complements. This group is referred to as the *complementary segment*. In group 2, there is a mass $1 - \lambda$ of identical consumers who view products A and B as independent goods. This group is referred to as the *independent segment*.[13]

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[12]On the contrary, Condorelli and Padilla (2020a,b) study the effects of "privacy policy tying", the enveloper's strategy of linking its privacy policies in the origin and target markets to extract the user's consent to the combination of data generated in both markets for commercial purposes.

[13]In Section 7, we provide an alternative modeling of the coexistence of complementary and independent groups based on the degree of complementarity of the two goods.

The market for tying product A (e.g., app markets, Internet portals or online market-places, premium credit cards) is monopolized by firm 1. The tying good is produced at constant marginal cost c_A, which is normalized to zero without loss of generality. All the consumers have the same reservation value u for the tying good. Each consumer in the market yields an (average) advertising profit of $\alpha \geq 0$ on the other side of the tying-good market.[14] In the market for tied product B (e.g., search engines, mobile payment services, department stores/malls), there is competition between firm 1's product $B1$ and firm 2's product $B2$. Both firms have the same constant marginal cost $c_B \geq 0$. Consumers have reservation values v_1 and v_2 for products $B1$ and $B2$, respectively. Assume $v_2 \geq v_1 > c_B$; in other words, the rival's tied good is superior to the monopolist's in terms of product quality. Let $\Delta = v_2 - v_1$ denote the difference in consumer valuation for the two tied goods. The per-customer (average) advertising profit on the other side of the tied-good market is $\beta_1 > 0$ for group 1 consumers and $\beta_2 > 0$ for group 2 consumers, and the two advertising profits can differ (i.e., $\beta_1 \neq \beta_2$). All the consumers have unit demand for either product $B1$ or product $B2$ (i.e., they single-home in market B). Multi-homing cases are considered later. For simplicity, we assume away fixed costs for all firms.

Unlike previous works such as Amelio and Jullien (2012) and Choi and Jeon (forth-coming), we allow for below-cost or negative prices in both markets. Firms cannot price discriminate between the two groups of consumers because of personal arbitrage or gov-ernment regulation. The consumers in the complementary segment have unit demand for a system consisting of two complements A and B, and their value for the system is given by $u + v_i, i = 1, 2$. We consider two distinct cases depending on the demand structure of group 2 consumers: either standalone demand for product B or independent demand for both products A and B.

3   Standalone demand for product B

Suppose all the consumers in the independent segment have standalone demand for prod-uct B only and no demand for product A (i.e., $u < 0$ due to disposal or maintenance

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[14]We may allow α to differ between the two groups. This change only affects firm 1's decision on whether to serve group 1 consumers only or both groups of consumers and, therefore, does not alter the qualitative results of the analysis.

costs).[15] Assume $\lambda > 0$ to make the analysis meaningful. The same setting is analyzed by Whinston (1990, Subsection III. C) in the context of the leverage theory of tying in one-sided markets with scale economies and a tying commitment, which are assumed away in our model without fixed costs. We explore a different role of the coexistence of the independent market in making tying profitable in two-sided markets.[16]

3.1 Separate sales

Given the perfect complementarity of group 1 consumers' demand for the goods, firm 1 will try to extract the rent generated in the tied-good market using a price squeeze. Specifically, firm 1 can squeeze firm 2's price using the price of product $B1$ (even though its inferior product $B1$ will not be sold) and capture part of the advertising profit generated in the tied-good market and the surplus created by the efficient rival through its pricing of monopolized product A. As usual, there are multiple equilibria in this pricing game. A price squeeze works only if firm 2 is active, making non-negative profits by selling product $B2$ in both markets at $p_{B2} = p_{B1} + \Delta$ for firm 1's price p_{B1}.[17] Without price discrimination between the two groups, this requires that[18]

$$\Pi_2(p_{B1}) \equiv \lambda(p_{B1} + \Delta + \beta_1 - c_B) + (1 - \lambda)(p_{B1} + \Delta + \beta_2 - c_B) \geq 0.$$

This break-even constraint for firm 2 sets a lower bound on firm 1's price for product $B1$:

$$p_{B1} \geq c_B - \Delta - \lambda\beta_1 - (1 - \lambda)\beta_2.$$

We assume that firm 1 can squeeze firm 2's price down to the point at which firm 2 just breaks even, thus allowing for the maximum possible profit for firm 1 under separate sales. In this sense, our approach is conservative in evaluating firm 1's tying incentive. Given $p_{B2}^e = c_B - \lambda\beta_1 - (1 - \lambda)\beta_2$, the maximum price firm 1 can charge for its monopolized product A is given by

$$p_A^e = u + v_2 - p_{B2}^e = u + v_1 + \Delta + \lambda\beta_1 + (1 - \lambda)\beta_2 - c_B,$$

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[15] We assume that the monopolist does not wish to provide subsidies to the consumers in the independent segment since they do not use product A even if they have one and therefore are not exposed to advertising.

[16] Hagiu, Julllien, and Wright (2020) use a similar setup to examine a multiproduct firm's incentive to host an efficient rival in the non-core product market.

[17] We assume that consumers buy superior product $B2$ when they are indifferent.

[18] It would be difficult for firms to distinguish consumers based on their perception of the complementarity of the two products.

and thus firm 1's profit under no tying is

$$\Pi_1^* = \lambda[u + v_1 + \alpha + \Delta + \lambda\beta_1 + (1 - \lambda)\beta_2 - c_B].$$

Firm 2's profit is squeezed to zero. The equilibrium prices of the tied goods are below cost and possibly negative when β_1 and β_2 are large.[19]

Total consumer surplus is

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\begin{aligned}
CS^* &= \lambda[u + v_2 - p_A^e - p_{B2}^e] + (1 - \lambda)[v_2 - p_{B2}^e] \\
&= (1 - \lambda)[v_2 + \lambda\beta_1 + (1 - \lambda)\beta_2 - c_B].
\end{aligned}
$$

The surplus of group 1 consumers is fully extracted through the pricing of the tying good, while group 2 consumers with standalone demand for product B enjoy a positive surplus thanks to the price squeeze. Social welfare is maximized with all the consumers using superior product $B2$:

$$W^* = \lambda[u + v_2 + \alpha + \beta_1 - c_B] + (1 - \lambda)[v_2 + \beta_2 - c_B].$$

3.2 Tying

Suppose firm 1 decided to bundle products A and $B1$. Facing identical consumers, firm 1 will not sell products A and $B1$ separately along with the bundle (i.e., no incentive for mixed bundling). Clearly, firm 1 will not sell product A separately if it intends to sell the bundle to group 1 consumers. Further, firm 1 has no strict incentive to sell product $B1$ separately since doing so will only reduce firm 2's profit without affecting its own

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[19]This equilibrium involves firm 1 pricing product $B1$ below cost and making no sales. One may wonder if this is reasonable, given that if the price of product $B2$ turns out to be slightly higher than expected (maybe due to trembling by firm 2), firm 1 will lose money by selling in the independent segment. To avoid such a loss, firm 1 needs to set the price of product $B1$ such as $p_{B1} \geq c_B - \beta_2$. With this pricing strategy, however, firm 1 has to forgo the profit gain that would be realized when firm 2 abides by $p_{B2}^e = c_B - \lambda\beta_1 - (1 - \lambda)\beta_2$. There is no other strategy of firm 1 to weakly dominate the fully squeezed price. The equilibrium strategy profile with the full price squeeze survives the notion of trembling-hand perfection. In an earlier version of this paper, we analyzed the case in which firm 1's price squeeze is limited to $p_{B1} = c_B - \beta_2$ and found that the results are qualitatively the same in all aspects. Obviously, in this case, firm 1's profit incentive for tying increases since its optimal profit under separate sales is smaller when the price squeeze is limited.

profit.[20] Further, firm 1 will not sell the bundle to group 2 consumers, even if they could disassemble the bundle and dispose of product A at no cost.[21]

Under the single-homing assumption, firm 2 is naturally excluded from the complementary segment under tying. Therefore, firm 1 can charge monopoly price $P_T = u + v_1$ for the bundle and earn profits of

$$\Pi_1^T = \lambda[u + v_1 + \alpha + \beta_1 - c_B].$$

Under tying, firm 1 can capture all the advertising profit (β_1) generated in the complementary segment. Without a price squeeze, however, it can no longer extract the efficiency gain (Δ) created by firm 2 and the advertising profit (β_2) generated in the independent segment. Given $P_T = u + v_1$ and no mixed bundling by firm 1, firm 2 can freely set monopoly price v_2 for product $B2$ and earn profits of $\Pi_2^T = (1 - \lambda)[v_2 + \beta_2 - c_B]$ from group 2 consumers. Tying here leads to a sort of price collusion, in which firms 1 and 2 each charges the monopoly price for the bundle and product $B2$, respectively. This reminds us of Carbajo, Meza, and Seidman (1990) and Chen (1997), who show that tying can relax price competition by market segmentation when consumers have different preferences for either the tying or the tied good. In our model, however, all the consumers are homogeneous and the collusive outcome comes from the coexistence of the two separate consumer groups with complementary and independent demand for the products.

Consumer surplus is fully extracted under price collusion:

$$CS^T = 0.$$

Tying involves a welfare loss since group 1 consumers in the complementary segment end up with the bundle containing inferior product $B1$:

$$W^T = \lambda[u + v_1 + \alpha + \beta_1 - c_B] + (1 - \lambda)[v_2 + \beta_2 - c_B].$$

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[20]A monopolist can use mixed bundling for price discrimination when consumers have heterogeneous preferences for the goods (see Schmalensee (1984) and McAfee, McMillan, and Whinston (1989) among others).

[21]Given that firm 2 is willing to lower the price of product $B2$ to $c_B - \beta_2$, the maximum price firm 1 can charge for the bundle is given by

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\begin{aligned}
v_1 - P_T \quad &\geq \quad v_2 + \beta_2 - c_B \\
&\implies \quad P_T = c_B - \Delta - \beta_2,
\end{aligned}
$$

which is less than the marginal production cost of the bundle, c_B.

3.3 Incentive for tying and its welfare effects

Comparing the equilibrium profits, we find that tying is profitable if and only if

$$\Pi_1^T > \Pi_1^* \Longrightarrow \beta_1 > \Delta + \lambda\beta_1 + (1 - \lambda)\beta_2.$$

Obviously, firm 1 has no tying incentive if only group 1 consumers exist in the market (i.e., $\lambda = 1$). Following the traditional Chicago argument, firm 1 can extract both efficiency gain Δ and advertising profit β_1 via a price squeeze under separate sales, and thus tying only reduces firm 1's profit by preventing this rent extraction. This logic may not hold in the presence of group 2 consumers with standalone demand for product B. If β_1 is greater than β_2, firm 1 cannot capture all the advertising profit (β_1) created in the complementary segment since the price squeeze is limited to $c_B - \Delta - \lambda\beta_1 - (1 - \lambda)\beta_2$ because of the break-even constraint of firm 2.[22] This imperfect rent extraction due to the presence of standalone demand for the tied good is the main source of the tying incentive here, which differs from those found in the literature (inability to extract surplus from future upgrades in Carlton and Waldman (2012); failure to extract the rent in the tying-good market in Etro and Caffarra (2017); an incomplete rent extraction due to non-negative price constraints in Choi and Jeon (forthcoming)).

Proposition 1 *With a strictly positive mass of standalone demand for the tied good (i.e., $\lambda < 1$), tying is profitable if*

$$\beta_1 > \beta_2 + \frac{\Delta}{1 - \lambda}, \tag{1}$$

and tying in this case reduces both consumer surplus and total welfare.[23]

For tying to be profitable, it is necessary for consumers with complementary demand to be more responsive to advertising than those with independent demand in the tied-good market. When β_1 is sufficiently larger than β_2, in the face of an imperfect rent extraction, the monopolist wishes to capture the large advertising profit β_1 generated in the complementary segment directly using tying, even if it has to forgo extracting efficiency

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[22]On the contrary, if β_2 is larger than β_1, under separate sales firm 1 can extract not only the advertising profit (β_1) generated in the complementary segment but also some of the advertising profit (β_2) generated in the independent segment and the extra surplus created by firm 2 (Δ) using a price squeeze.

[23]Assuming firm 1 sets $p_{B1} = c_B - \beta_2$ to avoid the loss incurred by firm 2's tremble with a high price, tying would be profitable only if $\beta_1 > \beta_2$.

gain Δ. Tying would be profitable only if $\beta_1 > \beta_2$ in the absence of the efficiency gain ($\Delta = 0$).

Not surprisingly, tying is more likely to be profitable, as the difference in the advertising profit between the complementary and independent segments ($\beta_1 - \beta_2$) is larger and the size of the efficiency gain (Δ) is smaller. Further, the incentive for tying increases as the relative size of the complementary segment declines (i.e., λ is smaller). This is because it is more difficult for firm 1 to extract β_1 via a price squeeze when the size of standalone demand is larger.[24] Tying is unprofitable if the tied-good market is one-sided (i.e., $\beta_1 = \beta_2 = 0$). However, the two-sidedness of the tying-good market is not necessary for the profitability of tying. This result is expected given that the tying incentive here is driven by the desire to capture the large extra profits generated in the tied-good market. Commitment to tying is not necessary either, since the profitability of tying does not depend on whether the rival firm is active. Finally, tying would not be profitable if the firms could discriminate the tied-good price between the two groups (see the Appendix for the proof). With price discrimination, the monopolist could extract all the rent created in the tied-good market via a price squeeze.

Group 2 consumers enjoy a positive surplus under separate sales, while the consumer surplus is fully extracted under tying because of price collusion. Hence, tying reduces consumer surplus:

$$CS^* = (1 - \lambda)[v_2 + \lambda\beta_1 + (1 - \lambda)\beta_2 - c_B] > CS^T = 0.$$

This result contrasts with that obtained by Choi and Jeon (forthcoming), who find that tying benefits consumers in the independent-goods case and at least does not harm them in the perfect-complements case. Tying reduces total welfare as well since the consumers in the complementary segment are induced to use the bundle containing inferior product $B1$:

$$\Delta W \equiv W^T - W^* = -\lambda\Delta < 0.$$

---

[24]This result is similar to that obtained in Whinston's (1990) alternative use of the tied-good model, where the profitability of tying increases as the standalone demand for the tied good rises (i.e., the model setup becomes more like the pure independent-products case). However, the motivation for tying is different between the two models: the exclusion of rivals in Whinston and the extraction of extra profits on the other side of the tied-good market in ours.

3.4 Multi-homing

Suppose consumers can multi-home in the tied-good market (i.e., they can purchase product $B2$ together with the bundle if they wish). Assume that multi-homers use only superior product $B2$ (i.e., they do not use product $B1$ contained in the bundle). Consequently, it is firm 2 that can extract the advertising profit generated from multi-homers in market B. A proportion $\theta \in (0, 1]$ of consumers in each group can multi-home at no cost; however, the remainder cannot multi-home because of the cost of carrying additional products.[25] The separate-sales equilibrium is not affected by multi-homing since all the consumers use superior product $B2$ in the equilibrium. The possibility of multi-homing only decreases firm 1's tying profit and, therefore, tying is never profitable with multi-homing if it is unprofitable with single-homing.

Let us examine how multi-homing affects the tying equilibrium. Only group 1 consumers can multi-home. Hence, it is meaningless to sell the bundle to group 2 consumers, as they will not use product A and so no advertising profits would be generated. No firm will set a price higher than the single-homing equilibrium price since then demand would be zero for the firm. Below, we show that the price of the bundle is the same as the single-homing equilibrium price $u + v_1$ and the price of product $B2$ is set as $\Delta = v_2 - v_1$, which is lower than the single-homing equilibrium price v_2. Firm 2 lowers the price of product $B2$ to attract multi-homers in the complementary segment. Group 1 consumers will multi-home if

$$u + v_1 - P_T \leq u - P_T + v_2 - p_{B2}.$$

Hence, a necessary condition for multi-homing to occur is $p_{B2} \leq \Delta$.

The multi-homing decision is independent of the bundle price. Thus, the optimal bundle price is simply given as $u + v_1$. Likewise, firm 2's optimization problem is independent of the bundle price:

$$\max_{p_{B2}} : (p_{B2} + \beta_1 - c_B)\lambda x + (p_{B2} + \beta_2 - c_B)(1 - \lambda), \ x = \begin{cases} 0 \text{ if } p_{B2} > \Delta \\ \theta \text{ if } p_{B2} \leq \Delta \end{cases}$$

Multi-homing occurs for $p_{B2} \leq \Delta$. In this case, the optimal price is $p_{B2} = \Delta$ and firm 2 earns profits of $(\Delta + \beta_1 - c_B)\lambda\theta + (\Delta + \beta_2 - c_B)(1 - \lambda)$. On the contrary, for $p_{B2} > \Delta$, multi-homing does not occur, and firm 2 chooses the monopoly price v_2 and earns profits

---

[25]One may introduce consumer heterogeneity in terms of the multi-homing cost, but this only complicates the analysis without changing the results qualitatively.

of $(v_2 + \beta_2 - c_B)(1 - \lambda)$. To focus on the case in which multi-homing occurs, we assume that firm 2's profits are larger with multi-homing:

$$\beta_1 > \frac{(1 - \lambda)v_1}{\lambda\theta} - \Delta + c_B, \tag{2}$$

which holds if λ and β_1 are sufficiently large. Then, the equilibrium prices are $P_T^M = u + v_1$ and $p_{B2}^M = \Delta$ and the mass $\lambda\theta$ of group 1 consumers will multi-home. Firm 1's profit is

$$\Pi_1^{TM} = \lambda[u + v_1 + \alpha + \beta_1(1 - \theta) - c_B],$$

which is smaller than the profit obtained under single-homing by the amount of advertising revenue generated from the multi-homers of mass $\lambda\theta$.

Comparing firm 1's profits in the two regimes, we find that tying is profitable if

$$\theta < 1 - \lambda \text{ and } \beta_1 > \frac{(1 - \lambda)\beta_2 + \Delta}{1 - \lambda - \theta}. \tag{3}$$

The tying profit is smaller with multi-homing but tying can still be profitable if $\theta < 1 - \lambda$ and β_1 is sufficiently large.

The profitability of tying requires the proportion of multi-homers to be sufficiently small. It would be pointless to use tying to extract advertising profits with a large number of multi-homers. Thus, a positive mass $\lambda(1 - \theta)$ of group 1 consumers always use inferior product $B1$ in the bundle and, therefore, profitable tying reduces total welfare even with multi-homing. However, tying may increase consumer surplus with multi-homing. Group 1 consumers' surplus is fully extracted in both regimes irrespective of multi-homing. Group 2 consumers' surplus may be larger under tying than separate sales because of the reduced price of product $B2$. Specifically, under Assumption (2), the price of product $B2$ is $c_B - \lambda\beta_1 - (1 - \lambda)\beta_2$ under separate sales and Δ under tying. Hence, tying increases consumer surplus if $c_B - \lambda\beta_1 - (1 - \lambda)\beta_2 > \Delta$ and decreases it otherwise. Therefore, tying is profitable and raises consumer surplus with multi-homing if

$$\theta < 1 - \lambda \text{ and } \frac{(1 - \lambda)\beta_2 + \Delta}{1 - \lambda - \theta} < \beta_1 < \frac{c_B - (1 - \lambda)\beta_2 - \Delta}{\lambda},$$

which holds, for example, when $v_1 = 1, \beta_2 \to 0, \Delta \to 0$ and $c_B > 1/\theta$ and $\lambda > 1/2$ (i.e., the price reduction of product $B2$ due to multi-homing is sufficiently large).

Proposition 2 *Multi-homing occurs and tying is profitable if $\theta < 1 - \lambda$ and β_1 is sufficiently large. Profitable tying reduces total welfare but may raise consumer surplus.*

4 Independent demand for products A and B

Now, suppose that group 2 consumers have independent unit demand for products A and B, respectively.

4.1 Separate sales

Firm 1 can choose whether to sell its monopolized product A only to group 1 consumers or to all the consumers in both groups. When selling product A only to group 1 consumers, firm 1 will try to capture the rent created in the market via a price squeeze. As before, we assume that firm 1 can squeeze firm 2's price up to the point at which firm 2 just breaks even. The equilibrium outcome is the same as that obtained in the standalone demand model. Given $p_A^e > u$, group 2 consumers indeed do not purchase product A, validating partial participation. The only difference is that we now need to consider group 2 consumers' consumption of product A to evaluate total welfare. To avoid notational confusion, we relabel firm 1's profit, consumer surplus, and total welfare as $\Pi_1^*(1)$, $CS^*(1)$, and $W^*(1)$, respectively.

When selling product A to both groups of consumers, the maximum price firm 1 can charge is the monopoly price $p_A^m = u$ (i.e., a price squeeze is not feasible for group 2 consumers with independent demand) Here, firm 1 gives up extracting the extra surplus generated in the tied-good market. Given $p_A^m = u$, price competition in market B yields $p_{B1}^e = c_B - \lambda\beta_1 - (1-\lambda)\beta_2$ and $p_{B2}^e = c_B - \lambda\beta_1 - (1-\lambda)\beta_2 + \Delta$. We assume that all the consumers buy product $B2$ at these prices. Then, firm 1's profit is

$$\Pi_1^*(2) = u + \alpha,$$

and firm 2's profit is $\Pi_2^*(2) = \Delta$. Consumer surplus is

$$CS^*(2) = v_1 + \lambda\beta_1 + (1-\lambda)\beta_2 - c_B,$$

which is positive given that $v_1 > c_B$. All the consumers enjoy a positive surplus due to the price competition in market B. Total welfare is maximized with all the consumers purchasing product A and superior product $B2$:

$$W^*(2) = u + \alpha + v_2 + \lambda\beta_1 + (1-\lambda)\beta_2 - c_B.$$

Which of the partial- and full-participation equilibria is realized depends on the relative size of the two groups. Firm 1 will serve both groups when the size of the independent segment is sufficiently large (i.e., λ is small).

Lemma 1 *There exists a critical value $\widehat{\lambda} \in (0,1)$ such that $\Pi_1^*(1) \geq \Pi_1^*(2)$ for $\lambda \geq \widehat{\lambda}$ and $\Pi_1^*(1) \leq \Pi_1^*(2)$ for $\lambda \leq \widehat{\lambda}$.*

Tying

Suppose firm 1 decides to bundle products A and $B1$. As before, it has no incentive to sell products A or $B1$ separately along with the bundle. Firm 1 can choose between selling the bundle only to group 1 consumers and selling it to both groups.[26]

When selling the bundle to group 1 consumers only, the equilibrium outcome is the same as that obtained in the standalone demand model, except for group 2 consumers being excluded from the consumption of product A. Tying leads to a collusive outcome where firm 1 charges $u + v_1$ for the bundle and firm 2 charges v_2 for product $B2$. We denote firm 1's profit, consumer surplus, and total welfare in this case by $\Pi_1^T(1)$, $CS^T(1)$, and $W^T(1)$ respectively.

When selling the bundle to both groups of consumers, the equilibrium prices are determined by the price competition between the bundle and standalone product $B2$ in the independent segment. Assume u is sufficiently large that firm 1 wins this competition. Since firm 2 is willing to lower its price to $c_B - \beta_2$, the maximum price firm 1 can charge for its bundle is given by

$$
\begin{aligned}
u + v_1 - P_T(2) &\geq v_2 + \beta_2 - c_B \\
&\implies P_T^e(2) = u - \Delta - \beta_2 + c_B,
\end{aligned}
\tag{4}
$$

which is lower than the optimal price $u + v_1$ under partial participation by $v_2 + \beta_2 - c_B$. Instead, firm 1 can capture all the advertising profits generated in both groups. Firm 1's profit is

$$
\Pi_1^T(2) = u + \alpha - \Delta + \lambda(\beta_1 - \beta_2),
$$

and firm 2 earns zero profits. Assume α and/or β_1 are large so that for $\Pi_1^T(2) > 0$ even if $P_T^e(2) < 0$. All the consumers enjoy a positive net surplus because of the price competition in the independent segment:

$$
CS^T(2) = v_1 + \Delta + \beta_2 - c_B > 0,
$$

---

[26]The option of selling the bundle to group 2 consumers only is not feasible without price discrimination since the competition between the bundle and standalone product $B2$ would yield an equilibrium price of the bundle that is lower than group 1 consumers' willingness to pay.

given that $v_1 > c_B$. Tying thus results in a welfare loss since all the consumers use inferior product $B1$:

$$W^T(2) = u + v_1 + \alpha + \lambda\beta_1 + (1 - \lambda)\beta_2 - c_B.$$

The relative profitability of the partial- and full-participation equilibria depends on the magnitudes of $u + \alpha$ and Δ. When Δ is larger than $u + \alpha$, firm 1 always chooses to sell the bundle only to group 1 consumers because it is too costly to compete against superior product $B2$. If Δ is smaller than $u + \alpha$, however, firm 1 sells the bundle to group 1 consumers only when λ is large and to both groups when λ is small.

Lemma 2 *i) If $u + \alpha > \Delta$, there exists a critical value $\widetilde{\lambda} \in (0, 1)$ such that $\Pi_1^T(1) \geq \Pi_1^T(2)$ for $\lambda \geq \widetilde{\lambda}$ and $\Pi_1^T(1) \leq \Pi_1^T(2)$ for $\lambda \leq \widetilde{\lambda}$. ii) If $u + \alpha \leq \Delta$, $\Pi_1^T(1) \geq \Pi_1^T(2)$ for all $\lambda \in (0, 1)$.*

One of two contrasting competitive effects of tying—intensifying competition à la Whinston (1990) and relaxing competition à la Carbajo, Meza, and Seidman (1990)—is realized depending on the relative size of the two groups.

Incentive for tying

We first state some preliminary results on the profitability of tying. If either the complementary or the independent group exists alone, the monopoly profit under separate sales cannot be smaller than the profit under tying. According to the Chicago logic, tying only prevents the extraction of the extra surplus created by efficient rivals in the case of perfect complements ($\lambda = 1$) and forces the monopolist to bear the burden of competing against efficient rivals in the case of independent goods ($\lambda = 0$). The same logic applies to two-sided markets provided below-cost pricing (including negative prices) is allowed, as confirmed by Choi and Jeon (forthcoming). Since the monopoly profit is continuous in λ, the following limiting result is immediate from the above argument.

Lemma 3 *Tying is not profitable when $\lambda \to 0$ or $\lambda \to 1$.*

Tying is not profitable if the advertising profit created from the tied good is larger in the independent segment than in the complementary segment. This is not surprising given that the main motivation for tying here is to extract the large advertising profit β_1 generated in the complementary segment, as shown below. Further, tying is unprofitable if the market for the tied good is one-sided ($\beta_1 = \beta_2 = 0$).

Lemma 4 *Tying is not profitable if $\beta_1 \leq \beta_2$.*

Hence, for tying to be profitable, the two consumer groups must coexist and the advertising profit from the tied good must be larger in the complementary segment than in the independent segment. Below, we examine the profitability of tying when these two conditions hold.

Proposition 3 *Suppose that $\lambda \in (0,1)$ and $\beta_1 > \beta_2$. For $u + \alpha > \Delta$, it is possible that $\lambda \lesseqqgtr \widehat{\lambda}$ depending on parameter values, and tying is profitable if*

$$
\begin{cases}
\beta_1 > \beta_2 + \frac{\Delta}{1-\lambda} & \text{for } \max\{\widehat{\lambda}, \widetilde{\lambda}\} < \lambda < 1 \text{ (partial} \rightarrow \text{partial)} \\
\beta_1 > \beta_2 + \frac{\Delta}{\lambda} & \text{for } 0 < \lambda < \min\{\widehat{\lambda}, \widetilde{\lambda}\} \text{ (full} \rightarrow \text{full)} \\
\beta_1 > \frac{(1-\lambda)(u+\alpha)}{\lambda} - (v_1 - c_B) & \text{for } \widetilde{\lambda} < \lambda < \widehat{\lambda} \text{ (full} \rightarrow \text{partial)} \\
\beta_1 > \frac{2-\lambda}{1-\lambda}\beta_2 + \frac{\lambda(v_2 - c_B) + \Delta}{\lambda(1-\lambda)} - \frac{u+\alpha}{\lambda} & \text{for } \widehat{\lambda} < \lambda < \widetilde{\lambda} \text{ (partial} \rightarrow \text{full)}
\end{cases}
\tag{5}
$$

For $u + \alpha \leq \Delta$, firm 1 always sells the bundle only to group 1 consumers, and tying is profitable if

$$
\begin{cases}
\beta_1 > \beta_2 + \frac{\Delta}{1-\lambda} & \text{for } \widehat{\lambda} < \lambda < 1 \text{ (partial} \rightarrow \text{partial)} \\
\beta_1 > \frac{(1-\lambda)(u+\alpha)}{\lambda} - (v_1 - c_B) & \text{for } 0 < \lambda < \widehat{\lambda} \text{ (full} \rightarrow \text{partial)}
\end{cases}
\tag{6}
$$

For $u + \alpha > \Delta$, one of the following four cases can be realized as the equilibrium depending on the relative size of the two consumer groups:

(i) $\max\{\widehat{\lambda}, \widetilde{\lambda}\} < \lambda < 1$: Partial participation occurs in both regimes when the customer mass is sufficiently larger in the complementary segment than the independent segment. As in the standalone demand case, firm 1 suffers from an imperfect rent extraction under separate sales when β_1 is large (due to the rival's break-even constraint), and thus wishes to use tying to directly capture the large advertising profit β_1 generated in the complementary segment despite not being able to extract efficiency gain Δ.

(ii) $0 < \lambda < \min\{\widehat{\lambda}, \widetilde{\lambda}\}$: Full participation is realized in both regimes when the mass of the independent segment is sufficiently larger than the complementary segment. The motivation of tying here is different from that identified in case (i). No price squeeze or rent extraction is involved under separate sales since the monopolist wishes to sell the tying good to both groups. Instead, price competition occurs between the bundle and firm 2's product $B2$ in the independent segment. With tying, firm 1 can directly capture advertising profits β_1 and β_2 in the complementary and independent segments.

According to the Chicago logic, tying is unprofitable since the monopoly profit is eroded due to price competition against the efficient rival. When β_1 is large, however, this competitive pressure is not fully carried over to the complementary segment because the reduction in the bundle price is limited to $\Delta + \beta_2$. That is, the advertising profit (β_1) generated in the complementary segment is partially insulated from the price competition in the independent segment. Then, tying becomes profitable if the gain from extracting the large β_1 is greater than the loss from the price reduction. The profit loss is $\Delta + \beta_2$ in both markets and the advertising profits add up to $\lambda\beta_1 + (1 - \lambda)\beta_2$. The gain is larger than the loss if $\beta_1 > \beta_2 + \frac{\Delta}{\lambda}$. Unlike the previous case of partial participation, the tying incentive here is independent of whether price discrimination is possible for the tied good since there is no price squeeze under separate sales with selling the tying good to both groups of consumers.[27]

(iii) $\widetilde{\lambda} < \lambda < \widehat{\lambda}$: Full participation occurs under separate sales and partial participation under tying. In this case, tying reduces demand for the tying good. With tying, firm 1 can capture consumer surplus for product $B1$ and the advertising profit created in the complementary segment $(\lambda(v_1 + \beta_1))$. By not selling product A to group 2 consumers, however, firm 1 has to forgo extracting consumer surplus for product A and its advertising profit generated in the independent segment $((1 - \lambda)(u + \alpha))$. Further, firm 1 has to bear additional costs (λc_B) under tying. Hence, tying can be profitable if β_1, v_1 and λ are large relative to u, α and c_B. If it is, firm 1 uses tying simply to capture the rents generated from the tied good in the complementary segment. Firm 2 also benefits from tying if $(1 - \lambda)(v_2 + \beta_2 - c_B) > \Delta$.

(iv) $\widehat{\lambda} < \lambda < \widetilde{\lambda}$: Partial participation occurs under separate sales and full participation under tying. This case is unique in that tying leads to a demand expansion for the tying good. When β_1 is large, firm 1 can extract only part of the rent generated in the complementary segment under separate sales. Under tying, firm 1 can fully capture advertising profits β_1 and β_2, even though it has to take the loss following price competition against superior product $B2$. Further, firm 1 has to give up efficiency gain Δ and bear higher production costs under tying. Tying is profitable if the advertising profit β_1 obtained in the complementary segment is sufficiently large to outweigh the profit loss from price competition and higher production costs.

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[27]The same is true in case (iii) in which full participation occurs and no price squeeze is involved under separate sales.

For the case of $u + \alpha \leq \Delta$, either case (i) or case (iii) occurs and the equilibrium results are the same as above.

Recall that in the standalone demand case, the profitability of tying was monotonically decreasing in λ. This is no longer true with group 2 consumers' separate demand for the tying good because firm 1 now wishes to sell it to both groups of consumers when λ is small. Without a price squeeze under separate sales, the Chicago logic for perfect complements does not apply and the logic for independent goods comes into effect instead. Hence, firm 1 has to bear the burden of inefficiency when competing against the efficient rival under tying. When λ is small, the loss from price competition is larger than the gain from capturing advertising profit β_1 in the complementary segment. Inspecting the conditions in (5) and (6), we find that the minimum value of β_1 for tying to profitable rises as λ approaches 0 or 1 from the middle. This means that all other things being equal, tying is more likely to be profitable when the two groups are more or less similar in size (see Figure 1). For instance, in case (i), to have partial participation in both regimes, λ must take a high value, whereas the profitability of tying requires a small λ. Similarly, in case (ii), to ensure full participation in both regimes, λ must be small; however, a large λ is required for tying to be profitable.

Example 1 *Suppose $u + \alpha = 1.1$, $v_1 = c_B = \beta_2 = 0$, $v_2 = 1$, thus $\Delta = 1$. Note that $u + \alpha > \Delta$ and $\beta_1 > \beta_2 = 0$. Two critical values are $\widehat{\lambda} = \frac{-2.1 + \sqrt{4.4\beta_1 + 4.41}}{2\beta_1}$ and $\widetilde{\lambda} = \frac{1}{11} \simeq 0.09$, which are represented by the dashed lines in Figure 1. Note that $\lambda \lesseqqgtr \widehat{\lambda}$ implies $\beta_1 \lesseqqgtr \frac{1.1 - 2.1\lambda}{\lambda^2}$. It can thus be shown that $\widehat{\lambda} > \widetilde{\lambda}$ for $\beta_1 < 110$ and $\widehat{\lambda} \leq \widetilde{\lambda}$ for $\beta_1 \geq 110$. For the case of $\beta_1 < 110$ (i.e., $\widehat{\lambda} > \widetilde{\lambda}$), tying is profitable if*

$$\begin{cases} \beta_1 > \frac{1}{1-\lambda} & \text{for } \widehat{\lambda} < \lambda < 1, \\ \beta_1 > \frac{1.1(1-\lambda)}{\lambda} & \text{for } \widetilde{\lambda} < \lambda < \widehat{\lambda}. \\ \beta_1 > \frac{1}{\lambda} & \text{for } 0 < \lambda < \widetilde{\lambda}, \end{cases} \tag{7}$$

Next, for the case of $\beta_1 > 110$ (i.e., $\widehat{\lambda} < \widetilde{\lambda}$) tying is profitable if

$$\begin{cases} \beta_1 > \frac{1}{1-\lambda} & \text{for } \widetilde{\lambda} < \lambda < 1, \\ \beta_1 > \frac{\lambda+1}{\lambda(1-\lambda)} - \frac{1.1}{\lambda} & \text{for } \widehat{\lambda} < \lambda < \widetilde{\lambda}, \\ \beta_1 > \frac{1}{\lambda} & \text{for } 0 < \lambda < \widehat{\lambda} \end{cases} \tag{8}$$

The region in which tying is profitable corresponds to the area above the U-shaped solid line in Figure 1, implying that tying is more likely to be profitable for intermediate values of λ than both extremes.
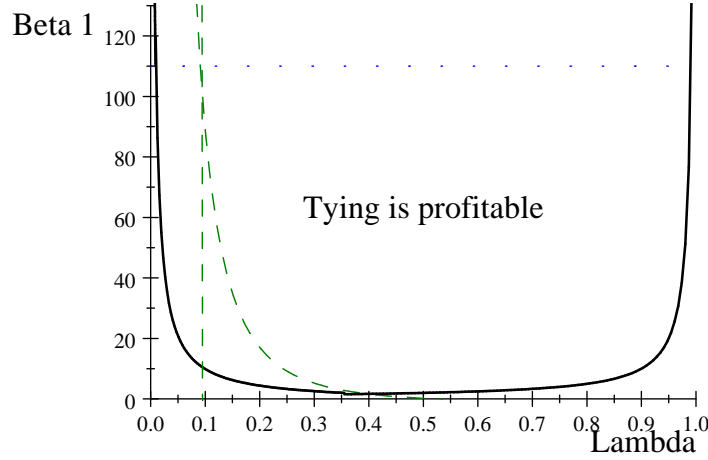
Figure 1: Areas in which tying is profitable

4.2   Welfare effects of tying

First, consider the case of $u + \alpha > \Delta$ (i.e., the extra surplus created by the rival firm is not that large). Suppose condition (5) is satisfied, so tying is profitable in all four cases.

(i) Partial participation in both regimes ($\max\{\widehat{\lambda}, \widetilde{\lambda}\} < \lambda < 1$): As we saw in the standalone demand case, tying reduces both consumer surplus and total welfare. Under tying, consumer surplus is fully extracted because of price collusion and group 2 consumers are induced to use inferior product $B1$.

(ii) Full participation in both regimes ($0 < \lambda < \min\{\widehat{\lambda}, \widetilde{\lambda}\}$): Tying reduces consumer surplus: $CS^T(2) - CS^*(2) = \Delta - \lambda(\beta_1 - \beta_2) < 0$ given that $\beta_1 > \beta_2 + \frac{\Delta}{\lambda}$ (the condition for tying to be profitable). Although all the consumers enjoy a positive surplus in both regimes, price competition is less intense under tying than separate sales for β_1 being larger than β_2. Tying also reduces total welfare by forcing all the consumers to use the bundle containing inferior product $B1$.

(iii) Full participation under separate sales and partial participation with tying ($\widetilde{\lambda} < \lambda < \widehat{\lambda}$): Consumer surplus is positive under separate sales because of price competition, while their surplus is fully extracted under tying with price collusion. Thus, tying reduces consumer surplus. Tying also reduces total welfare by inducing group 1 consumers to use inferior product $B1$ and excluding group 2 consumers from the consumption of product A as well.

(iv) Partial participation under separate sales and full participation with tying ($\widehat{\lambda} <$

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$\lambda < \widetilde{\lambda}$): In this case, tying induces group 2 consumers to newly purchase product A and this demand expansion has a positive effect on welfare. Meanwhile, group 1 consumers are forced to use inferior product $B1$ under tying. As a result, the welfare effect of tying depends on the relative strength of these two effects. The change in total welfare due to tying is calculated as

$$\Delta W = W^T(2) - W^*(1) = (1-\lambda)(u+\alpha) - \Delta.$$

Hence, if $\Delta > (1-\lambda)(u+\alpha)$, total welfare decreases under tying. In this case, the loss from consuming inferior product $B1$ is greater than the gain from expanding demand for product A. If $\Delta < (1-\lambda)(u+\alpha)$, however, the gain from the demand expansion outweighs the loss from using the inferior good and, therefore, total welfare increases under tying. We now examine how tying affects consumer surplus. Only group 2 consumers enjoy a positive surplus under separate sales, while all the consumers enjoy a positive surplus under tying. Therefore, the change in consumer surplus due to tying is

$$\Delta CS = CS^T(2) - CS^*(1) = \lambda[v_2 - c_B - (1-\lambda)\beta_1 + (2-\lambda)\beta_2].$$

If $\Delta > (1-\lambda)(u+\alpha)$, consumer surplus is definitely lower under tying than separate sales ($\Delta CS < 0$ given the profitability condition $\beta_1 > \frac{2-\lambda}{1-\lambda}\beta_2 + \frac{\lambda(v_2-c_B)+\Delta}{\lambda(1-\lambda)} - \frac{u+\alpha}{\lambda}$). If $\Delta < (1-\lambda)(u+\alpha)$, however, tying may increase or decrease consumer surplus (ΔCS can be either positive or negative depending on the parameter values). In particular, if v_2 and β_2 are large and β_1 and c_B are small, the gain in consumer surplus because of price competition under tying is larger than the surplus gain resulting from a price squeeze under separate sales. A large β_1 is required for tying to be profitable. Therefore, it is difficult to meet both conditions for tying to be profitable and for consumer surplus to increase at the same time. Nevertheless, it is still possible, as shown in the following example.

Example 2 *Suppose $u + \alpha = 1, c_B = \beta_2 = 0$ and $\Delta \to 0$ (i.e., $v_1 \simeq v_2$). Note that $\widehat{\lambda} = \frac{\sqrt{(1+v_2)^2+4\beta_1}-(1+v_2)}{2\beta_1} < \widetilde{\lambda} = \frac{1}{1+v_1} \simeq \frac{1}{1+v_2}$ is always satisfied. For $\widehat{\lambda} < \lambda < \widetilde{\lambda}$, partial participation under separate sales and full participation under tying are realized and tying is profitable if $\beta_1 > \frac{v_2}{1-\lambda} - \frac{1}{\lambda}$. Since $\Delta \simeq 0 < (1-\lambda)(u+\alpha) = 1-\lambda$ for $\lambda \in (0,1)$, tying increases total welfare in this case. Tying also increases consumer surplus if $\beta_1 < \frac{v_2}{1-\lambda}$ and decreases it otherwise. Thus, the condition for tying to be profitable and increase consumer surplus is $\frac{v_2}{1-\lambda} - \frac{1}{\lambda} < \beta_1 < \frac{v_2}{1-\lambda}$. On the contrary, profitable tying reduces consumer surplus*

when $\beta_1 > \frac{v_2}{(1-\lambda)}$. For example, for $v_2 = 2$, we have $\widehat{\lambda} = \frac{\sqrt{4\beta_1+9}-3}{2\beta_1}$ and $\widetilde{\lambda} = \frac{1}{3}$. Hence, $\widehat{\lambda} < \lambda < \widetilde{\lambda}$ corresponds to $\lambda < \frac{1}{3}$ and $\beta_1 > \frac{1-3\lambda}{\lambda^2}$ (the region above the dashed line in Figure 2). In this case, tying is always profitable in the relevant parameter range. Tying increases consumer surplus if $\beta_1 < \frac{2}{(1-\lambda)}$ and decreases it otherwise (see the two cases separated by the solid line in Figure 2). Hence, the case of profitable tying increasing both consumer surplus and total welfare is depicted by the region above the dashed line and below the solid line, and the case of increasing total welfare but decreasing consumer surplus is depicted by the region above the dashed and solid lines in Figure 2.
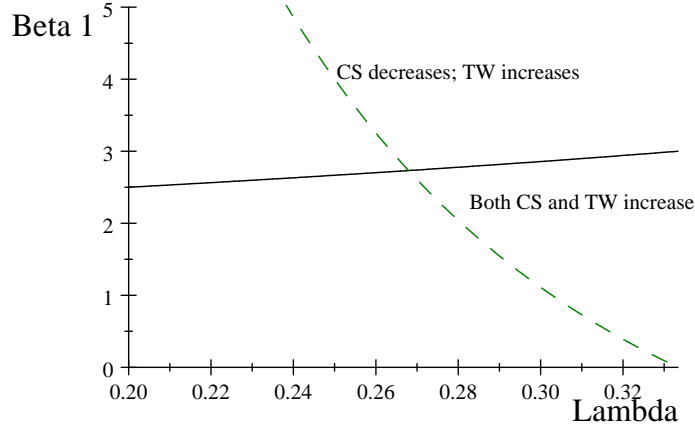

Figure 2: Welfare effects of tying

Next, consider the case of $u + \alpha \le \Delta$ (i.e., the extra surplus created by the rival is sufficiently large). Here, either partial or full participation can be realized under separate sales but partial participation always occurs under tying. In this case, tying always reduces consumer surplus and total welfare without a demand expansion.

The discussion so far is summarized in the following proposition. The demand expansion is necessary but not sufficient for tying to improve social welfare and consumer surplus.

Proposition 4 *Profitable tying reduces both consumer surplus and total welfare, except for the case of a demand expansion in which tying increases total welfare and may increase consumer surplus if $\Delta < (1 - \lambda)(u + \alpha)$.*

The above result implies that when the relative size of the complementary segment is sufficiently large so that partial participation occurs under separate sales, banning price

discrimination on tied goods may force monopolies that cannot extract a sufficient rent from separate sales to engage in tying that reduces consumer surplus and social welfare.

4.3 Multi-homing

As before, suppose that a proportion $\theta \in (0, 1]$ of consumers in each group can multi-home without cost and multi-homers use only superior product $B2$. The no-tying equilibria do not change with multi-homing. Let us examine how the tying equilibria are affected by multi-homing. First, consider the partial-participation equilibrium at which only group 1 consumers buy the bundle and group 2 consumers buy the standalone product $B2$. To ensure that group 2 consumers do not multi-home, the price of the bundle must be such that

$$
\begin{aligned}
v_2 - p_{B2} \quad &> \quad u - P_T + v_2 - p_{B2} \\
&\implies \quad P_T > u.
\end{aligned}
$$

In this case, firm 1 will set monopoly price $u + v_1$ for the bundle, earning profits of $\Pi_1^{TM}(1) = \lambda[u + v_1 + \alpha + \beta_1(1 - x) - c_B]$, where x is 0 or θ depending on whether group 1 consumers multi-home or not. For this to be an equilibrium, firm 1 should not have an incentive to deviate to a price lower than or equal to u. Suppose that $P_T \leq u$. Then, the mass $(1 - \lambda)\theta$ of group 2 consumers will buy the bundle together with product $B2$. In this case, firm 1 will choose $P_T = u$ and earn profits of $\Pi_1^{BM}(1) = \lambda[u + \alpha + \beta_1(1 - x) - c_B] + (1 - \lambda)\theta[u + \alpha - c_B]$. For firm 1 not to deviate, it must be that $\Pi_1^{TM}(1) > \Pi_1^{BM}(1)$,

$$
\frac{\lambda v_1}{(1 - \lambda)\theta} > u + \alpha - c_B, \tag{9}
$$

which requires a sufficiently large λ.

On the contrary, for group 1 consumers of mass $\lambda\theta$ to multi-home, the price of product $B2$ must be such that $p_{B2} \leq \Delta$, i.e., the price of product $B2$ must be smaller than or equal to efficient gain Δ. As before, firm 2's profit maximization problem is independent of the bundle price. Under Assumption (2), firm 2 optimally chooses $p_{B2} = \Delta$ and multi-homing occurs in the complementary segment. Hence, under Assumptions (2) and (9) a partial-participation equilibrium is realized with prices $P_T^M(1) = u + v_1$ for the bundle and $p_{B2}^M(1) = \Delta$ for product $B2$. Firm 1's equilibrium profit in this case is

$$
\Pi_1^{TM}(1) = \lambda[u + v_1 + \alpha + \beta_1(1 - \theta) - c_B],
$$

which is the same as that obtained in the standalone demand model.

Next, consider the full-participation equilibrium at which both groups of consumers buy the bundle and multi-home. Recall that for multi-homing to occur, it is required that $p_{B2} \leq \Delta$. Assume that β_1, β_2 and Δ are sufficiently large for firm 2 to make positive profits by selling product $B2$ to multi-homing consumers. To win the price competition in the independent segment, firm 1 has to choose its bundle price such that

$$
\begin{aligned}
u + v_1 - P_T \quad &\geq \quad v_2 - p_{B2} \\
&\implies \quad P_T - u + \Delta \leq p_{B2}.
\end{aligned}
$$

This, together with $p_{B2} \leq \Delta$, implies $P_T \leq u$. Then, firm 1's profit maximization problem can be written as

$$
\max_{P_T} : \begin{cases} (P_T + \alpha - c_B) + \lambda\beta_1(1 - \theta) + (1 - \lambda)\beta_2(1 - \theta), & \text{if } P_T - u + \Delta \leq p_{B2} \leq \Delta \\ \lambda(P_T + \alpha - c_B) + \lambda\beta_1(1 - \theta), & \text{if } p_{B2} < P_T - u + \Delta \end{cases} .
$$

Firm 2's profit maximization problem is

$$
\max_{p_{B2}} : \begin{cases} (p_{B2} + \beta_1 - c_B)\lambda\theta + (p_{B2} + \beta_2 - c_B)(1 - \lambda)\theta, & \text{if } P_T - u + \Delta \leq p_{B2} \leq \Delta \\ (p_{B2} + \beta_1 - c_B)\lambda\theta + (p_{B2} + \beta_2 - c_B)(1 - \lambda), & \text{if } p_{B2} < P_T - u + \Delta \end{cases} .
$$

For a high P_T, firm 2 may try to win the price competition against the bundle in the independent segment.

For $\theta < 1$, no pure-strategy price equilibria exist because of the discontinuity of the profit functions.[28] There is a pure-strategy equilibrium for $\theta = 1$, at which firm 1 chooses $P_T^M(2) = u$ and firm 2 chooses $p_{B2}^M(2) = \Delta$. In this case, firm 1's profit is

$$
\Pi_1^{TM}(2) = u + \alpha - c_B.
$$

There exists a cutoff value $\widetilde{\lambda}^M = \frac{u + \alpha - c_B}{u + v_1 + \alpha - c_B} \in (0, 1)$ such that $\Pi_1^{TM}(1) \geq \Pi_1^{TM}(2)$ if $\lambda \geq \widetilde{\lambda}^M$ and $\Pi_1^{TM}(1) \leq \Pi_1^{TM}(2)$ if $\lambda \leq \widetilde{\lambda}^M$.

Let us examine the tying incentive and its welfare effects with multi-homing. For $\theta < 1$, when λ is large, meaning that partial participation occurs in both regimes, the result is the same as that obtained in the standalone demand model. Tying becomes less profitable with multi-homing, but the tying incentive still exists if β_1 is sufficiently large.

---

[28] A rough proof is given in the Appendix. Mixed-strategy equilibria may exist, although we do not try to find one.

Total welfare is reduced but consumer surplus may increase under tying. Unfortunately, we cannot provide any result for λ being small so that only mixed-strategy equilibria may exist with full participation under tying. For $\theta = 1$, there exists a pure-strategy equilibrium even with full participation under tying. Four equilibrium configurations are possible depending on the values of $\widehat{\lambda}$ and $\widetilde{\lambda}^M$. After plugging in $\theta = 1$, firm 1's profits are

$$
\begin{aligned}
\Pi_1^*(1) &= \lambda[u + v_2 + \alpha + \lambda\beta_1 + (1 - \lambda)\beta_2 - c_B], \\
\Pi_1^*(2) &= u + \alpha
\end{aligned}
$$

under separate sales and

$$
\begin{aligned}
\Pi_1^{TM}(1) &= \lambda[u + v_1 + \alpha - c_B], \\
\Pi_1^{TM}(2) &= u + \alpha - c_B
\end{aligned}
$$

under tying. Not surprisingly, tying is not profitable in all four cases. With all the consumers multi-homing ($\theta = 1$), tying cannot capture the advertising profits generated in the tied-good market.

5 Product differentiation in the tied-good market

In this section, we consider the case in which consumers' preferences for products $B1$ and $B2$ are horizontally differentiated à la Hotelling, while the preference for product A is still homogeneous for all the consumers. To simplify the analysis and focus on the effect of product differentiation on the tying incentive, we assume away the quality difference between products $B1$ and $B2$ ($v_1 = v_2 = v$, i.e., $\Delta = 0$). The consumers in each group are uniformly distributed on the unit interval $[0, 1]$ and firms 1 and 2 are located at 0 and 1, respectively. If a consumer located at $x \in [0, 1]$ purchases product Bi, s/he gains the utility of $v - t|x - x_i|$ less the price s/he pays, where $t|x - x_i|$ measures the disutility due to the difference between the purchased product and his/her ideal product with $x_i \in \{0, 1\}, i = 1, 2$. Assume that v is sufficiently large so that all the consumers buy one unit of product $B1$ or product $B2$ in the equilibrium.

5.1 Separate sales

When selling product A only to group 1 consumers, firm 1 will choose the price of product $B1$ considering its effect on the price it can charge for monopolized product A. Given

p_{B1} and p_{B2}, the indifferent type is given by $\widehat{x} = \frac{1}{2} - \frac{p_{B1} - p_{B2}}{2t}$. Assume that u, v, and α are large relative to t, meaning that firm 1 wishes to serve all the consumers in the complementary segment. Then, the price of product A is determined at the level at which the utility of consumers of type $\widehat{x}$ is fully extracted and therefore it holds that

$$u + v - p_A - p_{B1} - tx^* = 0$$
$$\implies p_A(p_{B1}, p_{B2}) = u + v - \frac{t}{2} - \frac{p_{B1} + p_{B2}}{2}.$$

Substituting $p_A(p_{B1}, p_{B2})$ into firm 1's profit, we obtain

$$\pi_1(p_{B1}, p_{B2}) = \lambda(u + v + \alpha - \frac{t}{2} - \frac{p_{B1} + p_{B2}}{2}) + [p_{B1} + \lambda\beta_1 + (1-\lambda)\beta_2 - c_B](\frac{1}{2} - \frac{p_{B1} - p_{B2}}{2t}),$$

and firm 2's profit is

$$\pi_2(p_{B1}, p_{B2}) = [p_{B2} + \lambda\beta_1 + (1-\lambda)\beta_2 - c_B](\frac{1}{2} + \frac{p_{B1} - p_{B2}}{2t}).$$

The best responses are given by

$$p_{B1}^R(p_{B2}) = \frac{p_{B2} + (1-\lambda)t - \lambda\beta_1 - (1-\lambda)\beta_2 + c_B}{2},$$
$$p_{B2}^R(p_{B1}) = \frac{p_{B1} + t - \lambda\beta_1 - (1-\lambda)\beta_2 + c_B}{2},$$

and the equilibrium prices and profits are as follows:

$$p_A^*(1) = u + v - c_B + \lambda\beta_1 + (1-\lambda)\beta_2 - \frac{(3-\lambda)}{2}t,$$
$$p_{B1}^*(1) = t - \frac{2}{3}\lambda t + c_B - \lambda\beta_1 - (1-\lambda)\beta_2,$$
$$p_{B2}^*(1) = t - \frac{1}{3}\lambda t + c_B - \lambda\beta_1 - (1-\lambda)\beta_2,$$

$$\pi_1^*(1) = \lambda(u + v + \alpha + \lambda\beta_1 + (1-\lambda)\beta_2 - c_B) + \frac{(7\lambda^2 - 30\lambda + 9)}{18}t,$$
$$\pi_2^*(1) = \frac{(3-\lambda)^2}{18}t.$$

Even if $\Delta = 0$, firm 1 still wishes to squeeze firm 2's price to enable itself to charge a higher price for product A to group 2 consumers who buy products A and $B2$. Without product A, both firms would not deviate from the Nash equilibrium $p^e = t + c_B - \lambda\beta_1 - (1-\lambda)\beta_2$ as in the standard Hotelling model. In the present setup, firm 1 lowers the price

of product $B1$ below p^e since the profit gain from raising the price of product A in market A is first order, while the loss from the deviation in market B is only second order. This incentive for a price squeeze is captured by the smaller intercept of its reaction function relative to firm 2's. The equilibrium indifferent type is $\widehat{x}^*(1) = \frac{1}{2} + \frac{\lambda}{6}$, which is larger than $\frac{1}{2}$ as a result of the price squeeze. Here, the price squeeze is constrained by the very nature of product differentiation, not by the rival's break-even constraint as in the baseline model. When p_{B1} is lowered, the sales volume of product $B2$ decreases, and the profits firm 1 makes by selling product $B1$ in both groups are also reduced. Further, the incentive for the price squeeze relies on the extent of product differentiation in market B. If the two products were homogeneous ($t = 0$), firm 1 would simply sell product $B1$ to all the consumers in the complementary segment given that $\Delta = 0$. We assume that $v - p^*_{B1}(1) - t\widehat{x}^*(1) \geq 0$:

$$v - c_B + \lambda\beta_1 + (1 - \lambda)\beta_2 \geq \frac{(3 - \lambda)}{2}t, \qquad (10)$$

so that all group 2 consumers buy a unit of product B. This condition implies $p^*_A(1) \geq u$, i.e., nobody in group 2 is willing to buy product A, thus confirming partial participation in the consumption of product A.

When selling product A to both groups of consumers, firm 1 has to choose the price of product A as $p^*_A(2) = u$ (no price squeeze). Standard Hotelling competition takes place in market B with the two firms facing the same effective marginal cost $c_B - \lambda\beta_1 - (1 - \lambda)\beta_2$ reflecting the average advertising profit generated in the tied-good market. The equilibrium prices and profits are given as

$$
\begin{aligned}
p^*_A(2) &= u, \\
p^*_{B1}(2) &= p^*_{B2}(2) = t + c_B - \lambda\beta_1 - (1 - \lambda)\beta_2,
\end{aligned}
$$

$$
\begin{aligned}
\pi^*_1(2) &= u + \alpha + \frac{t}{2}, \\
\pi^*_2(2) &= \frac{t}{2}.
\end{aligned}
$$

The indifferent type is $\widehat{x}^*(2) = \frac{1}{2}$. To ensure that all the consumers buy a unit of product B, we assume that $v - p^*_{B1}(2) - t\widehat{x}^*(2) \geq 0$, i.e.,

$$v - c_B + \lambda\beta_1 + (1 - \lambda)\beta_2 \geq \frac{3}{2}t. \qquad (11)$$

This condition guarantees the satisfaction of condition (10).

We find that if $\beta_1 > \beta_2 - \frac{2}{9}t$, either partial or full participation can be realized in the equilibrium depending on λ. Otherwise, firm 1 always sells product A to both groups of consumers.

Lemma 5 *Suppose that condition (11) holds and $u + v + \alpha + \beta_2 - c_B - \frac{5}{3}t > 0$. For $\beta_1 \geq \beta_2 - \frac{2}{9}t$, there exists a critical value $\lambda^s \in (0, 1)$ such that $\pi_1^*(1) \geq \pi_1^*(2)$ for $\lambda \geq \lambda^s$ and $\pi_1^*(1) \leq \pi_1^*(2)$ for $\lambda \leq \lambda^s$. For $\beta_1 < \beta_2 - \frac{2}{9}t$, it holds that $\pi_1^*(1) \leq \pi_1^*(2)$ for all $\lambda \in [0, 1]$.*

5.2 Tying

Consider first the case of selling the bundle to group 1 only. Assume that

$$u + v + \alpha - c_B + \beta_1 \geq 2t. \tag{12}$$

Here, u, v, and α must be large and t must be small; hence, firm 1 wishes to sell the bundle to all group 1 consumers.[29] Then, firm 1 will set the price of the bundle as $P_T(1) = u + v - t$. Hotelling competition in the independent segment yields symmetric equilibrium prices $p_{B1}(1) = p_{B2}(1) = t + c_B - \beta_2$ for products $B1$ and $B2$. Unlike in the baseline model, here firm 1 can make a positive profit in market B since consumers' preferences are horizontally differentiated.[30] Thus, the two firms' equilibrium profits are

$$
\begin{aligned}
\pi_1^T(1) &= \lambda(u + v + \alpha + \beta_1 - c_B - t) + (1 - \lambda)\frac{t}{2}, \\
\pi_2^T(2) &= (1 - \lambda)\frac{t}{2}.
\end{aligned}
$$

A necessary condition for all the consumers in group 2 to buy either product $B1$ or product $B2$ is

$$v - c_B + \beta_2 \geq \frac{3}{2}t. \tag{13}$$

Next, consider the case of selling the bundle to both groups of consumers. Facing competition against product $B2$ in the independent segment, firm 1 has to lower the

---

[29]Suppose firm 1 sells the bundle to group 1 consumers of type $x \in [0, a], a \leq 1$. Then, it can charge $u + v - at$ for the bundle and make profits of $\pi^S(a) = \lambda a(u + v + \alpha + \beta_1 - c_B - at)$. Note that $\frac{d\pi^S(a)}{da}\big|_{a=1} = \lambda(u + v + \alpha + \beta_1 - c_B - 2t) \geq 0$ if $u + v + \alpha + \beta_1 - c_B \geq 2t$.

[30]This is true even if $\Delta > 0$, provided Δ is sufficiently small.

bundle price below $P_T(1) = u + v - t$. All the consumers in the complementary segment will buy the bundle, while the consumers in the independent segment can choose between the bundle and standalone product $B2$. We focus on the case in which all the consumers in both groups buy the bundle in the equilibrium.[31] A necessary and sufficient condition for this to happen is[32]

$$u + \alpha \geq \frac{3 - \lambda}{1 - \lambda} t. \tag{14}$$

The consumers in the independent segment will buy the bundle if $u + v - P_T - tx \geq v - p_{B2} - t(1-x)$. Given that firm 2 is willing to lower its price to $c_B - \beta_2$, in order for all the consumers of $x \in [0, 1]$ to buy the bundle, firm 1 has to charge $P_T(2) = u + c_B - \beta_2 - t$ for it. Under condition (13), consumers of type $x = 1$ are willing to buy the bundle at this price. Then, firm 1's profit is given by

$$\pi_1^T(2) = u + \alpha + \lambda(\beta_1 - \beta_2) - t,$$

and firm 2 gets zero profits.

Lemma 6 *Suppose that conditions (12), (13) and (14) hold. Then, there exists a critical value $\lambda^t \in (0, 1)$ such that $\pi_1^T(1) \geq \pi_1^T(2)$ for $\lambda \geq \lambda^t$ and $\pi_1^T(1) \leq \pi_1^T(2)$ for $\lambda \leq \lambda^t$.*

5.3   Incentive for tying

Under the conditions specified above (including $\beta_1 \geq \beta_2 - \frac{2}{9}t$), both $\lambda^s \geq \lambda^t$ and $\lambda^s \leq \lambda^t$ are possible and therefore four different equilibria can be realized depending on the parameter values.[33] As before, tying is not profitable if either the complementary or the independent group exists alone and the following limiting result holds.

Lemma 7 *Tying is not profitable when $\lambda \to 0$ or $\lambda \to 1$.*

---

[31] If not, some consumers of high x will buy product $B2$ instead of the bundle or buy nothing in the equilibrium. We find that even in this case, the result does not change qualitatively.

[32] The equilibrium indifferent type when the two firms are active in the independent segment is $\widehat{x}^* = \frac{u+\alpha}{6t} + \frac{3-5\lambda}{6(1-\lambda)}$. Condition (14) is derived from $\widehat{x}^* \geq 1$.

[33] First, suppose $u + \alpha = 3, v = 1, \beta_1 = 10, \beta_2 = 1, c_B = 0$ and $t \leq \frac{3(1-\lambda)}{3-\lambda} \leq 1$. Then, all the required conditions are satisfied for $\lambda \in (0, 1)$. We obtain $\lambda^s = \frac{15t - 45 + 3\sqrt{25t^2 - 108t + 1197}}{7t + 162} < \lambda^t = \frac{6-3t}{10-3t}$ for all $t \in (0, 1)$ in this case. Next, suppose $u + \alpha = 3, v = 1, \beta_1 = 1.1, \beta_2 = 1, c_B = 0$ and $t \leq \frac{3(1-\lambda)}{3-\lambda}$. Assume further that $\lambda \leq \frac{3}{4}$ and thus $t \geq \frac{1}{3}$. Again, all the required conditions are satisfied for $\lambda \in (0, \frac{3}{4}]$ and $\lambda^s > \lambda^t$ holds for $t \in [\frac{1}{3}, 1]$.

Comparing firm 1's profits in the two regimes yields the following result.[34]

Proposition 5 *With Hotelling preferences for the tied good, tying is profitable if*

$$
\begin{cases}
\beta_1 > \beta_2 + \frac{(7\lambda-3)}{18(1-\lambda)}t & \textit{for } \max\{\lambda^s, \lambda^t\} < \lambda < 1 \textit{ (partial} \rightarrow \textit{partial)} \\
\beta_1 > \beta_2 + \frac{3}{2\lambda}t & \textit{for } 0 < \lambda < \min\{\lambda^s, \lambda^t\} \textit{ (full} \rightarrow \textit{full)} \\
\beta_1 > \frac{(1-\lambda)(u+\alpha)}{\lambda} - (v - c_B) + \frac{3}{2}t & \textit{for } \lambda^t < \lambda < \lambda^s \textit{ (full} \rightarrow \textit{partial)} \\
\beta_1 > \frac{(2-\lambda)\beta_2 + v - c_B}{1-\lambda} - \frac{u+\alpha}{\lambda} + \frac{(7\lambda^2 - 30\lambda + 27)}{18\lambda(1-\lambda)}t & \textit{for } \lambda^s < \lambda < \lambda^t \textit{ (partial} \rightarrow \textit{full)}
\end{cases}
\tag{15}
$$

Notice the similarity between the condition above and the condition (5) obtained in the baseline model. The only difference is the additional term containing t on the right-hand side of the inequalities instead of the term containing efficiency gain Δ (which is normalized to 0 here). In all four cases, tying is profitable if the advertising profit β_1 generated in the complementary segment is sufficiently large.

Consider the case of partial participation in both regimes. Under separate sales, firm 1 can capture only part of the extra surplus created from consumers of high x. Tying allows firm 1 to directly capture all the advertising profit β_1. With tying, however, firm 1 cannot benefit from firm 2's offering product $B2$ to consumers of high x. Tying is thus profitable if the former gain is larger than the latter loss.

For the case of full participation in both regimes, the bundle price is reduced because of competition against product $B2$ in the independent segment. Instead, firm 1 can capture advertising profits β_1 and β_2 generated from the tied-good users. Advertising profit β_2 in the independent segment is fully dissipated. However, advertising profit β_1 in the complementary segment, if it is large, is partially insulated from price competition. Firm 1 wishes to use tying if the gain from capturing large advertising profit β_1 outweighs the loss from price competition. A similar logic applies to the other two cases with intermediate values of λ. Not surprisingly, tying becomes more profitable as t falls, since the negative impact of tying on firm 1's profit is reduced as products $B1$ and $B2$ become more similar.

---

[34]The equilibrium of the standalone demand model is essentially the same as that in the partial-partial case in Proposition 5.

5.4 Welfare effects of tying

Suppose tying is profitable in all four cases (i.e., β_1 is sufficiently large that condition (15) is satisfied).

(i) Partial participation in both regimes ($\max\{\lambda^s, \lambda^t\} < \lambda < 1$): Group 1 consumers obtain a larger surplus under tying because price competition is stronger under tying with all group 1 consumers being served than under separate sales with a price squeeze being limited. Consumers in group 2, on the contrary, gain a larger surplus under separate sales because of firm 1's price squeeze. Therefore, the effect of tying on consumer surplus is unclear at first sight. Consumer surpluses under separate sales and tying are calculated as

$$
\begin{aligned}
cs^*(1) &= (1-\lambda)(v + \beta_2 - c_B) + \lambda(1-\lambda)(\beta_1 - \beta_2) - (\frac{17}{36}\lambda^2 - 2\lambda + \frac{5}{4})t, \\
cs^T(1) &= (1-\lambda)(v + \beta_2 - c_B) - \frac{(5-7\lambda)}{4}t.
\end{aligned}
$$

Thus, the change in consumer surplus due to tying is

$$
\Delta CS = \lambda[(1-\lambda)(\beta_2 - \beta_1) + \frac{(17\lambda - 9)}{36}t],
$$

which is always negative under the profitability condition $\beta_1 > \beta_2 + \frac{(7\lambda-3)}{18(1-\lambda)}t$ for all $\lambda \in (0, 1)$. Therefore, tying reduces consumer surplus in this case.

Some welfare losses are involved in both regimes. Under separate sales, consumers of type $x \in (\frac{1}{2}, \frac{1}{2} + \frac{\lambda}{6})$ in the two groups use the less preferred product $B1$. Under tying, consumers of type $x \in (\frac{1}{2}, 1]$ in group 1 use the bundle containing the less preferred product $B1$, while all the consumers in group 2 use their preferred product B. We can easily check that the welfare loss is greater under tying than separate sales. Hence, tying reduces total welfare as well.

(ii) Full participation in both regimes ($0 < \lambda < \min\{\lambda^s, \lambda^t\}$): Under separate sales, consumers benefit from Hotelling competition in market B and their surpluses add up to

$$
cs^*(2) = v + \lambda\beta_1 + (1-\lambda)\beta_2 - c_B - \frac{5}{4}t.
$$

Under tying, consumers benefit from the price competition between the bundle and product $B2$ and the resulting consumer surplus is

$$
cs^T(2) = v + \beta_2 - c_B + \frac{1}{2}t.
$$

Then, the change in consumer surplus due to tying is

$$\Delta CS = \lambda \left(\beta_2 - \beta_1 \right) + \frac{7}{4}t.$$

Given the profitability condition $\beta_1 > \beta_2 + \frac{3}{2\lambda}t$ for this case, consumer surplus increases under tying if $\beta_2 + \frac{3}{2\lambda}t < \beta_1 < \beta_2 + \frac{7}{4\lambda}t$ and decreases if $\beta_1 > \beta_2 + \frac{7}{4\lambda}t$. Unlike in the case of homogeneous preferences, tying may increase consumer surplus without expanding demand for the tying good.

Total welfare is maximized under separate sales with all the consumers purchasing product A and their preferred product B. Under tying, however, all the consumers buy the bundle containing product $B1$. Therefore, the transportation cost increases for consumers of type $x \in (\frac{1}{2}, 1]$ and social welfare is reduced due to tying.

(iii) Full participation under separate sales and partial participation under tying ($\lambda^t < \lambda < \lambda^s$): In this case, tying reduces the sales of product A. The change in consumer surplus due to tying is

$$\Delta CS = -\lambda(v + \beta_1 - c_B - \frac{7}{4}t),$$

which is negative under the profitability condition $v + \beta_1 - c_B > \frac{3}{2}t + \frac{1-\lambda}{\lambda}(u + \alpha)$ and condition (14) ($u + \alpha > \frac{3-\lambda}{1-\lambda}t$). Here, tying reduces consumer surplus by making price competition less intense than under separate sales. Total welfare decreases under tying since group 2 consumers are excluded from the consumption of product A and group 1 consumers of type $x \in (\frac{1}{2}, 1]$ purchase the bundle containing their less preferred product $B1$.

(iv) Partial participation under separate sales and full participation under tying ($\lambda^s < \lambda < \lambda^t$): In this case, tying expands demand for product A. The change in consumer surplus due to tying is

$$\Delta CS = (v - c_B)\,\lambda - \beta_1 \lambda \left(1 - \lambda \right) + \beta_2 \lambda (2 - \lambda) + \frac{(17\lambda^2 - 72\lambda + 63)}{36}t.$$

Given the profitable condition of tying, consumer surplus increases with tying if $\frac{(2-\lambda)\beta_2 + v - c_B}{1-\lambda} - \frac{u+\alpha}{\lambda} + \frac{(7\lambda^2 - 30\lambda + 27)}{18\lambda(1-\lambda)}t < \beta_1 < \frac{(2-\lambda)\beta_2 + v - c_B}{1-\lambda} + \frac{(17\lambda^2 - 72\lambda + 63)}{36\lambda(1-\lambda)}t$ and decreases if $\beta_1 > \frac{(2-\lambda)\beta_2 + v - c_B}{1-\lambda} + \frac{(17\lambda^2 - 72\lambda + 63)}{36\lambda(1-\lambda)}t$. The intuition behind the result is as follows. Under separate sales, group 2 consumers benefit from firm 1's price squeeze, while group 1 consumers' surpluses are extracted via the price squeeze. Under tying, all the consumers benefit from price competition and their surplus increases as β_2 is larger (i.e., the price of the bundle is reduced

more).[35] However, the profitability of tying requires a large β_1.[36] Hence, for tying to be profitable and to increase consumer surplus, it is necessary for β_1 to be large but not that large relative to β_2.

Tying raises the consumption of product A but induces more consumers to use the less preferred product $B1$. The positive effect of the demand expansion dominates the negative effect of inefficient consumption. The change in total welfare due to tying is

$$\Delta TW = (1 - \lambda)(u + \alpha) - \frac{(3 - \lambda)(3 + \lambda)}{36} t,$$

which is positive under condition (14) $(u + \alpha > \frac{3-\lambda}{1-\lambda} t)$. Hence, social welfare increases with tying and, not surprisingly, the increase in welfare is larger when t is small.

Proposition 6 *With horizontal differentiation for the tied good, profitable tying tends to reduce consumer surplus and total welfare. However, tying may increase consumer surplus when all the consumers buy the bundle under tying and it increases total welfare provided demand expands under tying.*

The following example illustrates some of the results obtained above.

Example 3 *Suppose that $u + \alpha = 3, v = 1, \beta_2 = 1, c_B = 0$ and $t = \frac{1}{3}$. To satisfy conditions (11) and $\beta_1 \geq \beta_2 - \frac{2}{9} t$ under separate sales as well as conditions (12), (13), and (14) under tying, it is required that $\lambda \leq \frac{3}{4}$ and $\beta_1 \geq \frac{25}{27}$. We obtain $\lambda^s = \frac{3\sqrt{2}\sqrt{486\beta_1 + 377} - 120}{54\beta_1 - 47}$ and $\lambda^t = \frac{5}{9}$. The dotted lines in Figure 3 divide the parameter space into four regions corresponding to the four possible equilibrium configurations. Tying is profitable if*

$$\begin{cases} \beta_1 > \frac{51 - 47\lambda}{54(1-\lambda)} & \max\{\lambda^s, \lambda^t\} < \lambda < 1 \text{ (case 1)} \\ \beta_1 > \frac{1 + 2\lambda}{2\lambda} & 0 < \lambda < \min\{\lambda^s, \lambda^t\} \text{ (case 2)} \\ \beta_1 > \frac{6 - 7\lambda}{2\lambda} & \lambda^t < \lambda < \lambda^s \text{ (case 3)} \\ \beta_1 > \frac{294\lambda - 47\lambda^2 - 135}{54\lambda(1-\lambda)} & \lambda^s < \lambda < \lambda^t \text{ (case 4)} \end{cases},$$

which corresponds to the region above the solid line in Figure 3. Tying is always profitable for case 4. Consumer surplus increases in case 2 if $\frac{1+2\lambda}{2\lambda} < \beta_1 < \frac{7+12\lambda}{12\lambda}$ and in case 4 if $\frac{294\lambda - 47\lambda^2 - 135}{54\lambda(1-\lambda)} < \beta_1 < \frac{7(9 + 36\lambda - 13\lambda^2)}{108\lambda(1-\lambda)}$, which correspond to the regions below the

---

[35] Note that $cs^T(2)$ is increasing in β_2.

[36] However, β_1 should not be so large for consumer surplus to be relatively larger under tying than under separate sales.

two dashed lines respectively. Otherwise, consumer surplus decreases under tying. Total welfare decreases under tying in all three cases, except for case 4 in which tying increases total welfare with a demand expansion.
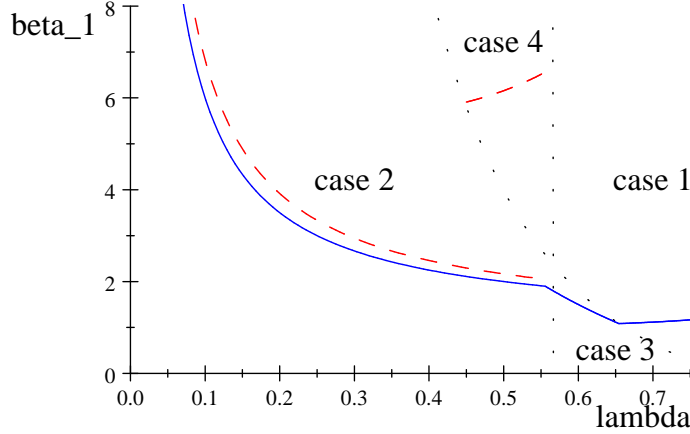

Figure 3: Tying with horizontal product differentiation

6 Application to real-world examples

Our model can be used to provide a theory of harm for the Google Android case in 2018. The European Commission concluded that Google has engaged in illegal tying by requiring mobile device manufacturers to pre-install Google apps such as Google Search and Chrome browser as a condition for licensing the Google app store (Google Play) through MADA (Mobile Application Distribution Agreement) contracts.[37] Of course, this practice does not necessarily exclude rivals from the app market under multi-homing. Recall, however, that our theory of tying encompasses a weak form of tying that does not involve exclusion of rivals as long as the monopoly's market share in the complementary segment of the tied-good market increases with tying. Google Play is almost essential for using Android-based mobile devices and therefore Google can be considered to be dominant in the Android app market. The markets for other applications such as search engines are, on the contrary, potentially competitive. Most apps possess a two-sided nature, and free services or rewards offered to app users (e.g., e-mail accounts and cloud

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[37]See the European Commission press release "Antitrust: Commission fines Google €4.34 billion for illegal practices regarding Android mobile devices to strengthen dominance of Google's search engine" on July 18, 2018.

storage) are examples of below-cost or negative pricing. In particular, in the Google Android case, app developers could make payments to manufacturers in the form of rebates in return for pre-installing their apps on their devices. These rebates should be effectively regarded as negative prices since the payments are partially passed onto final consumers in the fairly competitive device market. Consumers can be divided into two groups depending on the way of using apps: *mobile users* who normally use apps on a smartphone and view the two goods as perfect complements (complementary segment) and *desktop user*s who usually use apps on a desktop and use a mobile phone (if they have one) separately for communication (independent segment). Desktop users are willing to pass up buying a smartphone if it is too expensive, and would instead use a low-quality feature phone with limited access to the Internet. A consumer who uses a desktop more often because of age or work characteristics can be considered as a desktop user. As discussed in the Introduction, mobile users are typically more responsive to online advertising than desktop users ($\beta_1 > \beta_2$). Some rivals' apps are perceived to be superior to Google's ($\Delta \geq 0$). Google actually paid rebates to some device manufacturers, which implies a negative price for the bundle of Google Play and other apps. With this negative pricing, full participation is likely under tying. This is a sign that the relative size of the independent segment is large. Thus, full participation would have occurred under separate sales as well. With full participation in both regimes, Google's tying of its app store and other apps would have triggered price competition between the bundle and rivals' standalone apps, as reflected in the rebate competition for pre-installing apps. Advertising profit β_2 is fully dissipated because of price competition. However, the large advertising profit β_1 generated from mobile users is partly immune to this competitive pressure. It is thus perceivable that the extra revenue Google earns on the other sides of app markets (i.e., advertising profits from Google Search and YouTube) is larger than the rebates it actually paid to major OEMs of Android-based devices. If it is not, Google would not have paid such rebates given that the Android OS and apps are offered free of charge and their main source of revenue is search advertising on Android devices.[38] Thus, it is likely that Google found it profitable to bundle Google Play with other apps.

---

[38] The European Commission's fine of €4.34 billion was calculated on the basis of the value of Google's revenue from search advertising only in the EEA and it is less than 30% of the company's annual sales according to the Commission's 2006 Guidelines on fines. This provides an indication of the size of Google's actual advertising revenue. Unfortunately, we cannot provide a direct comparison because of the lack of data on the actual rebates paid to OEMs.

According to the earlier analysis, tying in this case usually reduces both consumer surplus and social welfare, which partly justifies the EU's decision on the Google Android case.

More specifically, consider the baseline model with independent demand for products A (app market) and B (other apps). Google's commitment to freely offer the bundle of Google Play and other apps imposes a price constraint on the bundle, $P_T \leq 0$. We find that tying equilibria with a negative price for the bundle can be realized only with full participation, where the price of the bundle and monopolist's profit are $P_T^e(2) = u - \Delta - \beta_2 + c_B \leq 0$ and $\Pi_1^T(2) = u + \alpha - \Delta + \lambda(\beta_1 - \beta_2)$, respectively.[39] For the monopoly tying profit under full participation to be larger than the profit under partial participation with $P_T = 0$, it is required that $\lambda < \frac{u + \alpha - \Delta}{\beta_2 + \alpha - c_B}$ (i.e., the relative size of the independent segment is not small). Hence, full participation is likely to occur under separate sales as well. The separate-sales equilibrium is not affected by the pricing constraint. Firm 1's profit in this case is given as $\Pi_1^*(2) = u + \alpha$. Tying is profitable if $\beta_1 > \beta_2 + \frac{\Delta}{\lambda}$ (i.e., β_1 is sufficiently large). Consumer surplus is $CS^*(2) = v_1 + \lambda\beta_1 + (1-\lambda)\beta_2 - c_B$ under separate sales and $CS^T(2) = v_1 + \Delta + \beta_2 - c_B$ under tying. The change in consumer surplus due to tying is $CS^T(2) - CS^*(2) = \Delta - \lambda\beta_1 + \lambda\beta_2 < 0$, which is negative given the profitability condition $\beta_1 > \beta_2 + \frac{\Delta}{\lambda}$. Hence, consumer surplus decreases with tying. Without an expansion in demand, total welfare also decreases under tying provided $\Delta > 0$.

Some early studies offered theories of harm related to the Google Android case. In Choi and Jeon (forthcoming), tying helps consumers coordinate on the bundle in the presence of strong intergroup network effects in the tied-good market. Here, the monopolist uses tying to extract the large extra revenue generated in the tying-good market (α).[40] Etro and Caffarra (2017), on the contrary, propose a theory of tying that allows Google to extract the extra value created by normal Android devices with Google Play over Android fork devices without Google Play (i.e., the difference in u) under the zero-price commitment. Both theories are concerned with the role of tying to extract the rent in the tying-good market. On the contrary, our model provides a logic for the anticompetitive tying used to capture the large extra revenue created on the other side of the tied-good market (i.e., the advertising profits generated from its killer apps such as Google Search and YouTube)

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[39] Both the partial- and the full-participation equilibria are feasible with the pricing constraint being binding (i.e., $P_T = 0$). However, we ignore these equilibria and focus on the case in which negative pricing of the bundle occurs in the form of rebates.

[40] This seems plausible given that Google collects fairly large transaction fees for the apps and in-app products offered through Google Play.

(β_1). Our theory of harm also differs from that suggested by Cornière and Taylor (2018) in which the purpose of tying is to ease rebate competition for device manufacturers.

A similar logic can be applied to the self-preferencing of search engines. A notable example is another Google case (2017) in which the European Commission fined Google for giving an advantage to its own online shopping service when presenting search results to users.[41] A similar allegation has been raised against NAVER, the leading Korean portal service provider.[42] Search engines and online shopping services are complements for online shoppers who search online and buy online. However, to offline shoppers who search online but buy offline, they are more like independent services. Search and online shopping services are usually free for consumers, whereas fees are charged to sellers for transactions taking place on the platforms. Online users tend to be more responsive to online advertising than offline users ($\beta_1 > \beta_2$). A theory of harm for such a practice can thus be constructed based on our model. A dominant search engine provider ties its search results to its own shopping service through self-preferencing to earn large extra revenue by collecting commissions from merchants for online transactions.

Also applicable is the practice used by online marketplaces, messengers, and portals (e.g., WeChat, Alibaba, Google, NAVER) to entice customers to use their own mobile payment services (WeChat Pay, Alipay, Google Pay, N Pay). In this case, tying takes place in the form of offering consumers monetary incentives to use their own payment service. For example, NAVER pays users a rebate of 2.5% of the down payment that can be used afterward only through N Pay, its own mobile payment service. Also, consumers are encouraged to use N Pay for online payments at NAVER Shopping, its own comparison shopping service. Consumers of online marketplaces and portals can be divided into two groups: heavy users who treat marketplaces and online payment services as perfect complements and light users who treat them as independent services. Heavy users produce larger extra revenue on the other side of the mobile payment service market than light users ($\beta_1 > \beta_2$). Dominant online marketplaces and portals can use tying to leverage their market power in online shopping to mobile payment services and thus collect large transaction fees from merchants using their mobile payment services.

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[41]See the European Commission press release "Antitrust: Commission fines Google €2.42 billion for abusing dominance as search engine by giving illegal advantage to own comparison shopping service" on June 27, 2017.

[42]See "Naver gets examiners' report from KFTC on dominance abuse through 'self-preferencing'" by LexisNexis on November 18, 2019.

Our last example is tying in the form of bundled discounts between premium credit card companies and department stores/shopping malls/duty-free shops. The free offer of various amenities and rewards at department stores can be regarded as below-cost pricing. Consumers are divided into two groups based on their income or store visit frequency. Users of premium cards who visit stores more frequently and generate larger sales than users of ordinary cards are more valuable to department stores since they can charge a high entrance or rental fee to merchants $(\beta_1 > \beta_2)$. A card company and retailers can jointly offer bundled discounts to encourage users of its premium card to buy products at designated places. In this way, related firms can extract the extra revenue generated on the other side of the two-sided market (e.g., merchant fees for the credit card company and slotting fees for department stores). Efficient rivals in both the credit card and the retail markets could be excluded under such bundled discounts.

7 An alternative modeling of the coexistence of complementary and independent groups

In the baseline model, individual consumers belong to either the complementary segment or the independent segment. In reality, however, dividing consumers into such a dichotomy might be difficult. Consumers may not know in advance whether they will consume two products as complements or as independent goods and, therefore, have to make purchasing decisions under uncertainty. A consumer's perception of a product's characteristics may also change over time or depending on the circumstances. In this section, we propose an alternative modeling of the coexistence of the two groups to reflect consumers' uncertainty about future consumption behaviors.

Suppose consumers are heterogeneous in their degree of uncertainty about the complementarity of products A and B. Let z denote the probability that a consumer treats the two products as perfect complements, where z is uniformly distributed on the interval $[0, 1]$. Assume that the advertising revenue generated from a consumer on the other side of market B is proportional to z. This is not necessary but is convenient for proving the profitability of tying later. We denote the average advertising profit generated from consumers whose type belongs to $[a, b]$ as $\overline{[a, b]}$. All the other elements are the same as in the baseline model.

Under separate sales, firm 1 will try to extract the rent via a price squeeze. Given p_A

and p_{B2}, the expected utility of a consumer with $z \in [0,1]$ is $u + v_2 - p_A - p_{B2}$ when buying products A and $B2$ and $(1-z)(v_2 - p_{B2})$ when buying product $B2$ (consuming product $B2$ alone is of no use with probability z). Buying product A alone is not an option to consumers since p_A is set larger than u under a price squeeze, as shown below. Let $\widehat{z}$ denote the type of consumers who are indifferent between the two options. As before, assume that firm 1 can squeeze firm 2's price until firm 2 breaks, i.e., $p_{B2} = c_B - \overline{[0,1]}$. Then, the following must hold for $\widehat{z}$:

$$
\begin{aligned}
u + v_2 - p_A - p_{B2} &= (1 - \widehat{z})(v_2 - p_{B2}) \\
&\Rightarrow p_A = u + \widehat{z}(v_2 - c_B + \overline{[0,1]}).
\end{aligned}
$$

Choosing p_A is equivalent to choosing $\widehat{z}$ for firm 1. Firm 1 will choose $\widehat{z}$ to maximize its profits $(1 - \widehat{z})[u + \alpha + \widehat{z}(v_2 - c_B + \overline{[0,1]})]$. A unique optimal value of $\widehat{z}$ (denoted $\widehat{z}^*$) exists since the profit function is concave given that $v_2 > c_B$.[43] Relabeling $\overline{(\widehat{z}^*, 1]} \equiv \beta_1$ and $\overline{[0, \widehat{z}^*]} \equiv \beta_2$, we can write $\overline{[0,1]} = (1 - \widehat{z}^*)\beta_1 + \widehat{z}^*\beta_2$. Then, firm 1's profit under separate sales can be written as

$$
\pi_1^{nt} = (1 - \widehat{z}^*)[u + \alpha + \widehat{z}^* v_2 + \widehat{z}^*(1 - \widehat{z}^*)\beta_1 + (\widehat{z}^*)^2 \beta_2 - \widehat{z}^* c_B].
$$

The functional form of the profit is similar to that obtained in the partial-participation equilibrium under separate sales in the baseline model (λ is replaced with $1 - \widehat{z}^*$). As in the baseline model, the rent extraction via a price squeeze is constrained by firm 2's break-even constraint.

Under tying, price competition occurs between the bundle and product $B2$, and a consumer of type z will buy the bundle if

$$
u + v_1 - (1 - z)(v_2 - p_{B2}) \geq P_T.
$$

Rather than solving for an equilibrium, we simply derive a profit (not necessarily optimal) firm 1 can obtain in equilibrium. To do this, consider the worst-case scenario for firm 1 by assuming that firm 2 will lower the price of product $B2$ to $p_{B2} = c_B - \overline{[0,1]}$. In this case, firm 1's profit will be $(1 - z)[u + v_1 - (1 - z)(v_2 - c_B + \overline{[0,1]}) + \alpha + \overline{(z,1]} - c_B]$. To ease the comparison of the equilibrium profits in the two regimes, let us suppose firm 1 has chosen $\widehat{z}^*$ (the optimal value of z under separate sales), which is usually different

---

[43]Firm 1 may wish to sell product A to all the consumers if u and α are sufficiently large relative to v_2 and $\overline{[0,1]}$. However, we focus on the case in which an interior solution is obtained.

from the optimal z that maximizes firm 1's profit under tying. Firm 1's profit in this case will be

$$\widehat{\pi}_1^t = (1 - \widehat{z}^*)[u + \alpha + \widehat{z}^* v_2 - \Delta + \widehat{z}^* (2 - \widehat{z}^*) \beta_1 - \widehat{z}^*(1 - \widehat{z}^*)\beta_2 - \widehat{z}^* c_B].$$

Note that $\widehat{\pi}_1^t > \pi_1^{nt}$ if $\beta_1 > \beta_2 + \frac{\Delta}{\widehat{z}^*}$, which constitutes a sufficient condition for tying to be profitable given that the true optimal profit of firm 1 must be larger than or at least equal to $\widehat{\pi}_1^t$. Thus, tying can be profitable if the advertising revenue obtained from consumers who are likely to treat the two products as complements is sufficiently large. Consumers of high z are induced to use inferior good $B1$ under tying. Thus, without a sufficient demand expansion, tying will reduce total welfare.

8   Conclusion

We proposed a theory of tying in two-sided markets in which firms can freely charge a price below cost or even a negative price. It was shown that the Chicago critique of the leverage theory of tying fails to hold when two consumer groups coexist in the market, one with complementary demand and the other with independent or standalone demand for the tying and tied goods. Two distinct tying mechanisms were found that can be used to increase monopoly profits. When the extra surplus created from the complementary segment for the tied good is sufficiently large, the monopolist may face an imperfect rent extraction because of the rival's break-even constraint under separate sales or the rent in the complementary segment is partly insulated from price competition under tying. In such situations, tying can be used to directly capture the large advertising profit created in the complementary market. The coexistence of the two groups with complementary and independent demand is crucial for the tying mechanisms to work. We found that such tying reduces consumer surplus and social welfare without expanding demand for the tying good. This result partially supports the theories of harm raised for some recent antitrust allegations such as the Google Android case. Finally, note that since the profitability of tying in our model does not requires the exclusion of rivals the intuition and results obtained can be applied to the practice of raising rivals' costs such as self-preferencing or requiring pre-installation as a default. The tying-good monopolist in a two-sided market has incentives for tying as long as it increases its own market share in the complementary segment of the tied-good market.

9   Appendix

No tying incentives with price discrimination for the tied good in the standalone demand model

Suppose that the two firms can charge different prices for product B based on the group identity of consumers. Then, firm 1 can fully extract both advertising profit β_1 and efficiency gain Δ by setting different prices for product $B1$ in the complementary and independent segments such as $p_{B1}^S = c_B - \Delta - \beta_1$ and $p_{B1}^I \geq c_B - \beta_2$, respectively. Firm 1's profit under separate sales is then

$$\Pi_1^{PD} = \lambda[u + v_1 + \alpha + \Delta + \beta_1 - c_B],$$

which is no smaller than the maximum profit under tying $\Pi_1^T = \lambda[u + v_1 + \alpha + \beta_1 - c_B]$ given that $\Delta \geq 0$. In fact, firm 1 could do better by lowering p_{B1}^S further and thereby inducing firm 2 to charge a lower price for product $B2$ in the complementary segment. With price discrimination, firm 1 can squeeze firm 2's price more effectively in the complementary segment using its price of product $B1$ in the independent segment. Hence, firm 1 does not opt for tying when price discrimination is possible for product B. The tying incentive would reappear if firm 1 could discriminate the bundle price under tying provided u is sufficiently large.

Proof of Lemma 1

Define

$$
\begin{aligned}
f(\lambda) &= \Pi_1^*(1) - \Pi_1^*(2) \\
&= (\beta_1 - \beta_2)\lambda^2 + (u + v_2 + \alpha + \beta_2 - c_B)\lambda - (u + \alpha).
\end{aligned}
$$

For $\beta_1 \neq \beta_2$, the quadratic function $f(\lambda)$ has two real roots:

$$\frac{-(u + v_2 + \alpha + \beta_2 - c_B) \pm \sqrt{(u + v_2 + \alpha + \beta_2 - c_B)^2 + 4(\beta_1 - \beta_2)(u + \alpha)}}{2(\beta_1 - \beta_2)}.$$

Note that $\sqrt{(u + v_2 + \alpha + \beta_2 - c_B)^2 + 4(\beta_1 - \beta_2)(u + \alpha)} > (u + v_2 + \alpha + \beta_2 - c_B)$. Thus, one negative root is eliminated and the critical value $\widehat{\lambda}$ corresponds to the other positive root:

$$\widehat{\lambda} = \frac{-(u + v_2 + \alpha + \beta_2 - c_B) + \sqrt{(u + v_2 + \alpha + \beta_2 - c_B)^2 + 4(\beta_1 - \beta_2)(u + \alpha)}}{2(\beta_1 - \beta_2)} > 0.$$

For $\beta_1 = \beta_2$, the linear equation $f(\lambda) = 0$ has the solution $\widehat{\lambda} = \frac{u+\alpha}{u+v_2+\alpha+\beta_2-c_B} \in (0,1)$. Given the assumption that $v_2 > c_B$, $f(0) = -(u+\alpha) < 0$ and $f(1) = \beta_1 + v_2 - c_B > 0$. Hence, there must exist $\widehat{\lambda} \in (0,1)$ such that $\Pi_1^*(1) \geq \Pi_1^*(2)$ for $\lambda \geq \widehat{\lambda}$ and $\Pi_1^*(1) \leq \Pi_1^*(2)$ for $\lambda \leq \widehat{\lambda}$.

Proof of Lemma 2

Define

$$
\begin{aligned}
g(\lambda) &= \Pi_1^T(1) - \Pi_1^T(2) \\
&= \lambda[u + v_1 + \alpha + \beta_2 - c_B] - [u - \Delta + \alpha].
\end{aligned}
$$

If $u + \alpha > \Delta$, we find $\widetilde{\lambda} = \frac{u-\Delta+\alpha}{(u+v_1+\alpha+\beta_2-c_B)} \in (0,1)$ such that $\Pi_1^T(1) \geq \Pi_1^T(2)$ for $\lambda \geq \widetilde{\lambda}$ and $\Pi_1^T(1) \leq \Pi_1^T(2)$ for $\lambda \leq \widetilde{\lambda}$. Note that $\widetilde{\lambda} < 1$ since $u + v_1 + \alpha + \beta_2 - c_B > u - \Delta + \alpha$ given that $v_1 > c_B$. If $u + \alpha \leq \Delta$, it holds that $g(\lambda) = \Pi_1^T(1) - \Pi_1^T(2) > 0$ for all λ given that $v_1 > c_B$.

Proof of Lemma 3

Note that $\lim_{\lambda \to 0} \Pi_1^*(1) = 0 < \lim_{\lambda \to 0} \Pi_1^*(2) = u + \alpha$ under no tying and $\lim_{\lambda \to 0} \Pi_1^T(1) = 0 < \lim_{\lambda \to 0} \Pi_1^T(2) = u - \Delta + \alpha$ under tying. We find that $\lim_{\lambda \to 0} \Pi_1^*(2) \geq \lim_{\lambda \to 0} \Pi_1^T(2)$ for $\Delta \geq 0$ with equality for $\Delta = 0$. Similarly, $\lim_{\lambda \to 1} \Pi_1^*(1) = u + v_1 + \alpha + \Delta + \beta_1 - c_B > \lim_{\lambda \to 1} \Pi_1^*(2) = u + \alpha$ under no tying and $\lim_{\lambda \to 1} \Pi_1^T(1) = u + v_1 + \alpha + \beta_1 - c_B > \lim_{\lambda \to 1} \Pi_1^T(2) = u - \Delta + \alpha + \beta_1 - \beta_2$ under tying given that $v_1 > c_B$ and $\Delta \geq 0$. Note that $\lim_{\lambda \to 1} \Pi_1^*(1) \geq \lim_{\lambda \to 1} \Pi_1^T(1)$ for $\Delta \geq 0$ with equality for $\Delta = 0$. Hence, firm 1 has no tying incentive in either case.

Proof of Lemma 4

Note that

$$
\Pi_1^T(1) = \lambda[u + v_1 + \alpha + \beta_1 - c_B] < \Pi_1^*(1) = \lambda[u + v_1 + \alpha + \Delta + \lambda\beta_1 + (1-\lambda)\beta_2 - c_B]
$$

and

$$
\Pi_1^T(2) = u - \Delta + \alpha + \lambda(\beta_1 - \beta_2) < \Pi_1^*(2) = u + \alpha
$$

for $\beta_1 \leq \beta_2$. Suppose that $\Pi_1^T(1) > \Pi_1^T(2)$. Since $\Pi_1^T(1) < \Pi_1^*(1)$ and $\Pi_1^T(1) \leq \max\{\Pi_1^*(1), \Pi_1^*(2)\}$, firm 1's profit must be lower under tying than under no tying. Next,

suppose that $\Pi_1^T(1) < \Pi_1^T(2)$. Similarly, the tying profit must be lower than the no-tying profit since $\Pi_1^T(2) < \Pi_1^*(2)$ and $\Pi_1^*(2) \leq \max\{\Pi_1^*(1), \Pi_1^*(2)\}$. Hence, firm 1's maximum profit under tying must be lower than its profit under no tying in any case.

Proof of no pure-strategy full-participation equilibria with multi-homing for $\theta < 1$ (A sketchy)

Firm 1's best response is $P_T^R = u - \Delta + p_{B2} - \varepsilon$ for all p_{B2}. Firm 2's best response: there exists a $\widehat{P}_T \leq u$ such that the optimal p_{B2} is Δ for $P_T \leq \widehat{P}_T$ and $P_T - u + \Delta - \varepsilon$ for $P_T \geq \widehat{P}_T$. There exists no intersection of the two best responses: when P_T is small, firm 2 wishes to raise p_{B2} to Δ. For $p_{B2} = \Delta$, firm 1 wishes to set P_T slightly lower than u. Then, firm 2 wishes to undercut firm 1's price to win the independent market. Such price cuts continue until firm 1 wishes to raise its price again to Δ (a price cycle).

Proof of Lemma 5

Define

$$
\begin{aligned}
h(\lambda) &= \pi_1^*(1) - \pi_1^*(2) \\
&= (\beta_1 - \beta_2 + \frac{7}{18}t)\lambda^2 + (u + v + \alpha + \beta_2 - c_B - \frac{5}{3}t)\lambda - u - \alpha.
\end{aligned}
$$

Note that $h(0) = -u - \alpha < 0$ and $h(1) = v + \beta_1 - c_B - \frac{23}{18}t > 0$ for $\beta_1 > \beta_2 - \frac{2}{9}t$ under condition (11). Given that $\beta_1 > \beta_2 - \frac{2}{9}t$, the quadratic function $h(\lambda)$ has two real roots. Eliminating one negative root, the critical value of λ corresponds to the other positive root:

$$
\lambda^s = \frac{\sqrt{\left(u+\alpha+v-c_B+\beta_2-\frac{5}{3}t\right)^2 + 4(u+\alpha)\left(\beta_1-\beta_2+\frac{7}{18}t\right)} - (u+\alpha+v-c_B+\beta_2-\frac{5}{3}t)}{2(\beta_1-\beta_2+\frac{7}{18}t)} \in (0,1),
$$

such that $\pi_1^*(1) \geq \pi_1^*(2)$ for $\lambda \geq \lambda^s$ and $\pi_1^*(1) \leq \pi_1^*(2)$ for $\lambda \leq \lambda^s$.

Proof of Lemma 6

Define

$$
\begin{aligned}
k(\lambda) &= \pi_1^T(1) - \pi_1^T(2) \\
&= (u + v + \alpha + \beta_2 - c_B - \frac{3}{2}t)\lambda - (u + \alpha - \frac{3}{2}t).
\end{aligned}
$$

Note that $k(0) = -(u + \alpha - \frac{3}{2}t) < 0$ given that $u \geq \frac{3-\lambda}{1-\lambda}t$ and $h(1) = v + \beta_2 - c_B > 0$ under the assumption $v > c_B$. We find $\lambda^t = \frac{u+\alpha-\frac{3}{2}t}{u+\alpha-\frac{3}{2}t+v+\beta_2-c_B} \in (0,1)$ such that $\pi_1^T(1) \geq \pi_1^T(2)$ for $\lambda \geq \lambda^t$ and $\pi_1^T(1) \leq \pi_1^T(2)$ for $\lambda \leq \lambda^t$.

Proof of Lemma 7

Note that $\lim_{\lambda \to 0} \pi_1^*(1) = \frac{1}{2}t < \lim_{\lambda \to 0} \pi_1^*(2) = u + \alpha + \frac{t}{2}$ under no tying and $\lim_{\lambda \to 0} \pi_1^T(1) = \frac{t}{2} < \lim_{\lambda \to 0} \pi_1^T(2) = u + \alpha - t$ under tying given that $u + \alpha \geq 3t$ (condition (14)). Since $\lim_{\lambda \to 0} \pi_1^*(2) = u + \alpha + \frac{t}{2} \geq \lim_{\lambda \to 0} \pi_1^T(2) = u + \alpha - t$ for $t > 0$, firm 1 has no tying incentive. Similarly, $\lim_{\lambda \to 1} \pi_1^*(1) = u + v + \alpha + \beta_1 - c_B - \frac{7}{9}t > \lim_{\lambda \to 1} \pi_1^*(2) = u + \alpha + \frac{t}{2}$ under no tying given that $v - c_B + \beta_1 - \frac{3}{2}t \geq 0$ (condition (11)) and $\lim_{\lambda \to 1} \pi_1^T(1) = u + v + \alpha + \beta_1 - c_B - t > \lim_{\lambda \to 1} \pi_1^T(2) = u + \alpha + \beta_1 - \beta_2 - t$ under tying given that $v_1 > c_B$ and $\Delta \geq 0$. Note that $\pi_1^*(1) = u + v + \alpha + \beta_1 - c_B - \frac{7}{9}t \geq \lim_{\lambda \to 1} \pi_1^T(1) = u + v + \alpha + \beta_1 - c_B - t$ for $t > 0$. Hence, firm 1 has no tying incentive in this case either.

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