Input price discrimination with differentiated final products*

Jong-Hee Hahn[†] and Chan Kim[‡]

April 12, 2018

Abstract

This paper examines the welfare effects of third-degree price discrimination by an input monopolist when downstream producers compete with differentiated goods and consumers have heterogeneous preferences for the products. The input monopolist's optimal pricing follows the standard inverse-elasticity rule, but its implication for welfare differs from the traditional analysis with homogeneous goods. Price discrimination can improve welfare even without an increase in total output or opening of new market. Also, the effect of price discrimination on consumer surplus differs from the one obtained for the case of price discrimination in final-goods markets. Our results shed new light on public policy regarding input price discrimination. We can no longer claim that price discrimination is harmful to society because it does not increase or reduces total output. Moreover, different policy responses are required depending on welfare standard. Simple policy guidelines are proposed that can be used in actual antitrust cases.

JEL Classification: L11

Keywords: third-degree price discrimination, intermediate goods market, product differentiation, competition policy, antitrust

*We thank Jaeok Park, Sang-Hyun Kim, Semin Kim, Jinwoo Kim and seminar participants at Yonsei and 2018 KAEA Annual Meeting for useful and insightful comments. All remaining errors are ours. This work was supported by the Ministry of Education of the Republic of Korea and the National Research Foundation of Korea (NRF-2016S1A3A2923769).

[†]Corresponding author: School of Economics, Yonsei University, Seoul 03722, South Korea. E-mail: hahnjh@gmail.com

[‡]School of Economics, Yonsei University, Seoul 03722, South Korea. E-mail: kchan3540@gmail.com

1 Introduction

The welfare effect of third-degree price discrimination has been an important topic in industrial organization and antitrust economics. A key insight from previous literatures is that price discrimination leads to a higher(lower) price to buyers with less(more) elastic demand and its welfare effect is closely related to the change in total output. It is well known since Pigou (1920) and Robinson (1933) that monopoly price discrimination in final-goods markets tends to reduce welfare if it does not increase total output. Schmalensee (1981) extended the result to nonlinear demands and proved that an increase in total output is necessary for price discrimination to increase welfare. Varian (1985) derived upper and lower bounds on the welfare change due to price discrimination with interdependent demands and increasing marginal cost. Schwartz (1990) generalized the analysis by eliminating the assumption of non-decreasing marginal cost. The basic intuition is that price discrimination leads to consumption distortion by inducing unequal marginal utilities across consumers and thus welfare has to be reduced if total output does not increase with price discrimination.[1]

Similar results are found in the third-degree price discrimination in intermediate-goods markets. An input monopolist facing downstream firms with different marginal costs has incentives to charge a higher input price to the low-cost firm and a lower input price to the high-cost firm. As shown by Katz (1987) and DeGraba (1990), such discriminatory pricing lowers social welfare by raising production costs for a given output.[2] Total output does not change with price discrimination in their models. Yoshida (2000) constructed a model in which total output can change with price discrimination and discovered that, somewhat surprisingly, the increase in total output of final goods is sufficient for welfare reduction.[3] A consensus in this line of research is that price discrimination involves distortion in consumption or production, and therefore in order for price discrimination to be welfare-improving there has to be some gain (e.g. an increase in total output) to offset the inefficiency created by the associated distortion.[4]

[1] The follow-on research includes Nahata, Ostaszewski and Sahoo (1990), Aguirre, Cowan and Vickers (2010), Cowan (2007, 2016, 2017) and others. Holmes (1989) extended the analysis to oligopolistic markets.

[2] DeGraba (1990) also pointed out the possibility that price discrimination distorts downstream firms' choice of production technology. In a somewhat different context, Katz (1987) showed that price discrimination can increase welfare by preventing inefficient backward integration. O'Brien (2014) showed that Katz's result can be reversed in a bargaining framework where an input monopolist negotiates the wholesale price with downstream firms. Kim and Sim (2015) showed that price discrimination can raise total output and welfare when the input monopolist contracts sequentially with downstream firms.

[3] In his model, the downstream firms differ not only in the marginal cost of other inputs but also in the efficiency in the use of the input supplied by the monopolist.

[4] Third-degree price discrimination can increase welfare if it opens a new market that would not be

Antitrust policy towards price discrimination has been mainly focused on intermediated-good markets. The primary US legislation covering price discrimination, the Robinson-Patman Act, was introduced with the intention of protecting small businesses against large buyers in intermediate-goods markets. So the major concern of the Robinson-Patman Act is harm to competition in the downstream market, so it does not directly applies to price discrimination practised against consumers in a final-good market. For example, an antitrust issue that has been given much attention recently is the FRAND obligation in the context of licensing of standard-essential patents. Standard-essential patents are patents that are declared by their owner as being necessary to implement a technical standard (OECD, 2014). Many standard setting organizations (SSOs) require members to commit to license patents essential to use of standards on fair, reasonable and non-discriminatory (so-call FRAND) terms. The third term "non-discriminatory" means that the patent holder should offer the same terms to all licensees, which corresponds to ban on input price discrimination.[5]

In reality, it is common for the same parts or technologies to be used to produce goods of different quality or location. The same set of mobile communications technologies such as LTE and 5G are licensed to all manufacturers producing mobile phones of different qualities. The same memory chip is used to produce electronic devices of different quality. To investigate the welfare effect of input price discrimination in such environments, we analyze stylized models where an input monopolist sells to two downstream firms competing with differentiated products. We first analyze the model of vertical(quality) differentiation and then proceed to the case of horizontal differentiation. Our exposition here is focused primarily on the case of quality differentiation, but similar results are obtained for the model of horizontal differentiation as will be shown later.

The input monopolist's optimal discriminatory pricing follows the standard inverse-elasticity rule, but its implication for welfare quite differs from the traditional analysis with homogeneous goods. In the homogeneous-goods models such as Katz(1987) and DeGraba (1990), the relative elasticity of input demand for downstream products depends solely on

served under uniform pricing. For instance, price discrimination always increases welfare and is in fact weakly Pareto-improving if it serves one of two independent markets that would be ignored under uniform pricing (see Tirole (1988, p.139)). Note that, however, this result is valid only in the case of two markets. If there exist more than two markets, market-opening price discrimination can improve welfare only under certain conditions, as shown by Kaftal and Pal (2008). See also Hausman and Mackie-Mason (1988) and Layson (1994) for the analysis on the welfare effect of price discrimination that opens new markets.

[5] The interpretation of "non-discriminatory" is still controversial. However, most tend to agree that "non-discriminatory" does not mean the patent holder should offer the same terms to all licensees in any circumstances whatsoever, and that it only requires similar treatment to similarly situated licensees (Sidak, 2013; Carlton and Shampine, 2013).

the equilibrium sales volumes. So a higher input price is charged for the firm with small marginal cost who faces more elastic demand with a larger sales volume, and conversely for the firm with large marginal cost. It is like taxing an efficient firm, thus raising the total cost of production for a given output and thereby reducing total welfare. This logic, however, does not always hold when downstream producers compete with differentiated products and consumers have heterogeneous preferences for the products. With vertical differentiation, the low-quality firm, due to the consumer participation constraint, may face a lower input price even if it has a larger sales volume than the high-quality firm. This is because the low-quality firm faces competition not only from the high-quality firm but also from consumers' no purchasing option, and therefore its demand is more sensitive to a price change compared with the high-quality rival who is insulated from the consumer participation constraint. This implies that the welfare result can be reversed, given that the sales volume tends to be positively correlated with the efficiency of the firms. Specifically, taxing a firm with less elastic input demand can improve welfare by inducing some consumers to switch to a socially more desirable product. Given that total output does not change with price discrimination in our model with linear demands, this result indicates that price discrimination can increase social welfare even if it does not involve an increase in total output or opening of new market. This result stands in stark contrast to those obtained in the previous literature.

The effect of price discrimination on consumer surplus is also quite different from previous works. We find that price discrimination makes all consumers weakly better off when the marginal production cost of the final good is sharply increasing with product quality. This is new to the literature on input price discrimination. Most of previous works, based on a Cournot model with homogeneous goods, failed to capture the distinct consumer surplus effect of input price discrimination under product differentiation. In our model, consumer surplus changes with price discrimination even though total output remains constant in the two pricing regimes. Our result also contrasts with that found in the analysis of price discrimination in final-goods markets. As shown by Cowan (2012), monopoly third-degree price discrimination in a final-good market usually lowers the surplus of consumers.

Also noteworthy is the fact that the effect of price discrimination on consumers is considerably different from the effect on total welfare and therefore different policy responses are required depending on welfare standard. If consumer surplus increases with price discrimination in final-good markets total welfare must increase as well. This property, however, no longer holds for price discrimination in intermediate-goods markets. Firm profits consist of the sum of the input monopolist's and downstream firms' profits, and thus if the downstream firms' profits fall significantly total welfare may decrease even if consumer surplus increases. So we need to treat the effects of price discrimination on consumer surplus and total welfare separately. When the marginal cost of the final good increases slowly in product quality, price discrimination reduces both consumer and total surplus and thus it is better to ban price discrimination in any case. When the marginal cost of the final good increases very sharply in product quality, price discrimination increases consumer surplus but decreases total welfare. In this case, price discrimination should be banned only under total welfare standard. Finally, in the intermediate case price discrimination increases total welfare but lowers consumer surplus, so a ban on price discrimination is requested only under consumer surplus standard.

We propose some policy guidances that can be used in real antitrust cases. Policymakers adopting consumer surplus as welfare standard are advised to check the ratio of input prices (that are easily observed in the relevant market) to see how consumers are affected by input price discrimination. On the other hand, the effect of price discrimination on total welfare can be assessed by observing profit margins or marginal costs of downstream firms.

In addition to the above-mentioned articles, there is a considerable amount of literature on input price discrimination. Inderst and Shaffer (2009) show that price discrimination by an input monopolist who can use two-part tariffs improves social welfare since it increases the efficiency of production by amplifying the cost difference between downstream competitors. However, this result may not hold if downstream firms are privately informed about their costs. Herweg and Müller (2014) find that the welfare effect of price discrimination in the presence of private information is ambiguous and depends on the degree of quantity distortion for the high-cost firm. Arya and Mittendorf (2010) consider an environment in which one downstream firm operates in multiple markets while the other operates only in a single market. They show that input price discrimination in this context may increase welfare by shifting output to markets with lower demand and less competition. Some authors studied the impact of input price discrimination on downstream entry. Herweg and Müller (2012) and Dertwinkel-Kalt, Haucap and Wey (2016) point out that price discrimination can intensify competition by fostering downstream entry while it may induces inefficient entries and increase production costs. Therefore, the net welfare effect depends on the relative strength of the two effects. Herweg and Müller (2016) investigated the same problem in the context of non-linear tariffs. On the other hand, Inderst and Valletti (2009) showed that price discrimination by an input monopolist, constrained by the threat of demand substitution, is harmful to consumers in the short run but tends to increase consumer surplus in the long run.

This paper differs from the existing literature in many respects. We examined the welfare effects of discriminatory input pricing when downstream producers compete with vertically or horizontally differentiated products and consumers are heterogeneous in their preferences for the products. To our knowledge, this is the first attempt to analyze the competitive effect of input price discrimination under spatial product differentiation. We have shown that the optimal input pricing under price discrimination follows the standard inverse-elasticity rule, but its welfare implications significantly differ from the previous works due to the asymmetric effect the consumer participation constraint has on the elasticities of the differentiated products. In particular, price discrimination can raise total welfare even without an increase in total output or opening of new market, which sharply contrasts with the conventional wisdom that says an increase in total output is necessary for total welfare to increase in order to offset the inefficiency created by distortion in consumption or production under price discrimination.[6] We find that with product differentiation input price discrimination can improve inframarginal consumption/production efficiency and so may be socially desirable even if it involoves some inefficiency at the margin. Also, the effect of price discrimination on consumer surplus is quite different from that on total welfare, and its pattern critically depends on the sensitivity of downstream marginal cost to product quality. These results give some important policy implications for price discrimination in intermediate-good markets. We should not presume that price discrimination is socially harmful just because it does not increase or reduces total output. Also, different policy responses are required depending on welfare standard. Finally, we propose simple policy guidelines that can be useful in dealing with actual antitrust cases.

2 Vertical differentiation

Consider an upstream monopolist selling an input or licensing a piece of technology to two downstream firms, each producing a good of quality $q_i, i = l, h$.[7] The quality levels of the products are exogenously given as $q_l < q_h$. Suppose the input is used in a fixed proportion (one-to-one) to produce a unit of the final good, irrespective of quality. The monopolist produces the input at constant marginal cost, which is normalized to zero for simplicity. Producing goods of quality q_i incurs constant marginal cost $c_i, i = l, h$, to the downstream firms, in addition to the cost of acquiring the input supplied by the monopolist. There is a

[6] Strictly speaking, there is a case where welfare would increase under price discrimination even if total output remains the same (Inderst and Shaffer, 2009). Note that, however, in their model total output actually increases in the price-discrimination equilibrium.

[7] Later we discuss what would happen if there were perfect (or Bertrand) competition for each of the two products.

continuum of consumers of mass 1 with unit demand for one of the vertically differentiated goods. A consumer of type θ obtains utility $u(\theta) = \theta q_i - p_i$ when purchasing a good of quality q_i at price p_i. Consumer type θ is uniformly distributed on the interval $[0, \overline{\theta}]$, where $\overline{\theta}$ is normalized to 1 without loss of generality.[8] The lower bound of consumer type is set to 0, which ensures partial participation in equilibrium. If the lower bound is larger than 0 we may have full-participation equilibria. This however does not change the qualitative results of the analysis but rather strengthens them, provided the lower bound is not too close to $\overline{\theta}$ so that both products are sold in equilibria.[9]

We consider the following two-stage game:

1. The upstream monopolist quotes (linear) input price w_i to the downstream firm producing goods of quality q_i.[10]

2. The downstream firms simultaneously and independently choose price p_i for the final good of quality q_i.

Given input prices w_l and w_h, the price equilibrium of the final-goods market is characterized as follows. Given p_l and p_h ($p_l < p_h$), two indifferent types are defined as

$$\widetilde{\theta} q_l - p_l = \widetilde{\theta} q_h - p_h \Rightarrow \widetilde{\theta} = \frac{p_h - p_l}{q_h - q_l}$$

and

$$\widetilde{\widetilde{\theta}} q_l - p_l = 0 \Rightarrow \widetilde{\widetilde{\theta}} = \frac{p_l}{q_l},$$

where the consumers of type $\theta \in [\widetilde{\widetilde{\theta}}, \widetilde{\theta}]$ buy the product of quality q_l, and those of type $\theta \in [\widetilde{\theta}, 1]$ buy the product of quality q_h. Thus, the demands for the low-quality and high-quality products are given by

$$D_l(p_l, p_h) = \widetilde{\theta} - \widetilde{\widetilde{\theta}} = \frac{p_h - p_l}{q_h - q_l} - \frac{p_l}{q_l}$$

and

$$D_h(p_l, p_h) = 1 - \widetilde{\theta} = 1 - \frac{p_h - p_l}{q_h - q_l}$$

respectively.

[8] We can always adjust quality as $q' \equiv \overline{\theta} q$ so that consumer type $\theta' \equiv \theta / \overline{\theta}$ is distributed on the unit interval $[0, 1]$.

[9] Increasing the lower bound is essentially the same as truncating some low types, and therefore it tends to reduce the demand of the low-quality product making it more elastic to price.

[10] It is assumed that the monopolist can distinguish two downstream firms producing goods of different qualities.

The firm l's profit-maximization problem is

$$\max_{p_l} : (p_l - w_l - c_l) \left(\frac{p_h - p_l}{q_h - q_l} - \frac{p_l}{q_l} \right),$$

which yields the reaction function $p_l^R(p_h) = \dfrac{q_l}{2q_h} p_h + \dfrac{w_l + c_l}{2}$.

Similarly, the firm h's problem is

$$\max_{p_h} : (p_h - w_h - c_h) \left(1 - \frac{p_h - p_l}{q_h - q_l} \right),$$

which gives the reaction function $p_h^R(p_l) = \dfrac{1}{2} p_l + \dfrac{(q_h - q_l) + w_h + c_h}{2}$.

From those two reaction functions, the equilibrium prices for the final goods are obtained as follows:

$$\begin{aligned} p_l^e &= \frac{q_l(q_h - q_l) + 2q_h(w_l + c_l) + q_l(w_h + c_h)}{4q_h - q_l}, \\ p_h^e &= \frac{2q_h(q_h - q_l) + 2q_h(w_h + c_h) + q_h(w_l + c_l)}{4q_h - q_l}. \end{aligned}$$

Then, the equilibrium demands of the two products can be written as

$$\begin{aligned} D_l^e &= \frac{q_h}{(4q_h - q_l)} - \frac{q_h(2q_h - q_l)(w_l + c_l)}{q_l(4q_h - q_l)(q_h - q_l)} + \frac{q_h(w_h + c_h)}{(4q_h - q_l)(q_h - q_l)}, \\ D_h^e &= \frac{2q_h}{(4q_h - q_l)} - \frac{(2q_h - q_l)(w_h + c_h)}{(4q_h - q_l)(q_h - q_l)} + \frac{q_h(w_l + c_l)}{(4q_h - q_l)(q_h - q_l)}. \end{aligned} \tag{1}$$

2.1 Optimal input prices

The one-to-one input-output ratio implies that the input demands for the two products are the same as those in (1) above. We will consider two pricing regimes: the first is the case where the monopolist can charge different input prices depending on the quality of the product (price discrimination) and the second is the case where the monopolist is constrained to charge the same input price regardless of quality (uniform pricing).

The following index, which denotes the cost-adjusted quality ratio of the two products, will be useful in the proceeding analysis:

$$\delta \equiv \frac{q_l - c_l}{q_h - c_h}.$$

It measures the relative cost efficiency of the low quality compared with the high quality. A high δ means that an increase in quality requires large marginal production costs, such as high-quality PCs being made of expensive high-performance CPU, memory chips and other parts. On the other hand, a low δ corresponds to the case where increasing quality involves is achieved mainly through fixed investments in R&D and little marginal costs are involved, as in software and information industries. When cost is proportional to quality, for instance $c_i = \lambda q_i$ ($\lambda < 1$), δ is identical to the quality or cost ratio that will be denoted $\delta_P \equiv \dfrac{q_l}{q_h} = \dfrac{c_l}{c_h} < 1$.

Price discrimination: The input monopolist chooses w_l and w_h to maximize profits:

$$\max_{w_l, w_h} : w_l D_l^e + w_h D_h^e.$$

Assuming a positive sales volume for both products, the optimal input prices are given as

$$w_l^* = \frac{q_l - c_l}{2}, w_h^* = \frac{q_h - c_h}{2}. \tag{2}$$

The condition for the high-quality firm to sell a strictly positive quantity, i.e. $\widetilde{\theta} < 1$, after plugging in the optimal input prices in (2), reduces to

$$\delta \equiv \frac{q_l - c_l}{q_h - c_h} < \frac{2q_h - q_l}{q_h} \equiv \delta_l.$$

Similarly, for the low-quality firm to sell a positive quantity, it must be that $\widetilde{\widetilde{\theta}} < \widetilde{\theta}$, i.e.[11]

$$\delta \equiv \frac{q_l - c_l}{q_h - c_h} > \frac{q_l}{2q_h - q_l} \equiv \delta_h.$$

Thus, it is required that $\delta_h < \delta < \delta_l$ for the validity of the optimal input prices in (2). If this condition does not hold, only a single product will be sold in equilibrium, the low-quality product for $\delta \geq \delta_l$ and the high-quality product for $\delta \leq \delta_h$ respectively.

Note that the ratio of the optimal input prices under price discrimination coincides with the cost-adjusted quality ratio of the two product, i.e.

$$\delta \equiv \frac{q_l - c_l}{q_h - c_h} = \frac{w_l^*}{w_h^*}.$$

As you can see later, this property is useful for implementing public policy toward input price discrimination.

Uniform pricing: The input monopolist chooses the same price w to maximize profits:

$$\max_{w} : w(D_l^e + D_h^e).$$

Assuming a positive sales volume for both products, the optimal (uniform) price is given by

$$w^* = \frac{q_l(q_h - c_h) + 2q_h(q_l - c_l)}{2(q_l + 2q_h)}, \tag{3}$$

[11] It always holds that $\widetilde{\widetilde{\theta}} = \frac{p_l^e}{q_l} > 0$.

Plugging in this optimal input price, the condition for the high-quality firm to sell a positive quantity (i.e., $\widetilde{\theta} < 1$) is

$$\delta \equiv \frac{q_l - c_l}{q_h - c_h} < \frac{8q_h^2 - q_h q_l - q_l^2}{6q_h^2} \equiv \overline{\delta}_l,$$

and the condition for the low-quality firm to sell a positive quantity (i.e., $\widetilde{\widetilde{\theta}} < \widetilde{\theta}$) is

$$\delta \equiv \frac{q_l - c_l}{q_h - c_h} > \frac{3q_h q_l}{2q_h^2 + 2q_h q_l - q_l^2} \equiv \overline{\delta}_h,$$

respectively. So the optimal input price in (3) is valid only if $\overline{\delta}_h < \delta < \overline{\delta}_l$. When it is violated, only the high-quality or the low-quality product will be sold in equilibrium.

2.2 Uniform pricing vs Price discrimination

We summarize the ordering of some important cutoff values of δ in the following lemma. From now on, we focus on the case of $\overline{\delta}_h < \delta < \overline{\delta}_l$, where both products are sold in a positive quantity under both pricing regimes.[12]

Lemma 1 $\delta_h < \delta_P < \overline{\delta}_h < 1 < \overline{\delta}_l < \delta_l$.

Comparing the optimal input prices in (2) and (3) yields the following proposition.

Proposition 1 $w_l^* \lesseqqgtr w^* \lesseqqgtr w_h^*$ *for* $\delta = \dfrac{q_l - c_l}{q_h - c_h} \lesseqqgtr 1$.

The relative size of the optimal input prices under the two pricing regimes depends on the value of the cost-adjusted quality ratio, $\delta = \frac{q_l - c_l}{q_h - c_h}$. When price discrimination is allowed, the input monopolist raises the price charged to the firm with a larger cost-adjusted quality while it reduces the price charged to the firm with a smaller cost-adjusted quality. Not surprisingly, this pricing rule is closely related to the elasticities of input demands of the two products. Plugging in $w_l = w_h = w$ into (1), we can rewrite the input demands for the two products in the following linear forms:

$$\begin{aligned} D_l^e(w) &= \alpha_l - \beta_l w, \\ D_h^e(w) &= \alpha_h - \beta_h w, \end{aligned}$$

[12] For δ being slightly higher than $\overline{\delta}_h$, the input monopolist may find it more profitable to serve only the high-quality firm than to serve both firms. Since this does not change the qualitative result of the analysis, we exclude this possibility by assuming that the input monopolist serves both downstream firms.

where

$$\begin{aligned}
\alpha_l &\equiv \frac{q_l q_h(q_h - q_l + c_h) - q_h(2q_h - q_l)c_l}{q_l(4q_h - q_l)(q_h - q_l)}, \\
\alpha_h &\equiv \frac{(2q_h - c_h)(q_h - q_l) - q_h(c_h - c_l)}{(4q_h - q_l)(q_h - q_l)}, \\
\beta_l &\equiv \frac{2q_h}{q_l(4q_h - q_l)}, \\
\beta_h &\equiv \frac{1}{(4q_h - q_l)}.
\end{aligned}$$

Both demands are downward sloping ($\beta_l > 0$ and $\beta_h > 0$) and the intercepts are positive given that $\alpha_l > 0$ and $\alpha_h > 0$ for $\delta \in (\delta_h, \delta_l)$. It is as if the input monopolist is practising third-degree price discrimination facing two separate markets, although the demands for the two products are interrelated via price competition.

It is well known that a discriminating monopolist charges a low(high) price in a market with more(less) elastic demand. For the linear demand $D_i^e(w) = \alpha_i - \beta_i w$ $(i = l, h)$, the price elasticity is given as

$$\varepsilon_i = -\frac{dD_i^e}{dw}\frac{w}{D_i^e} = \beta_i \frac{w}{\alpha_i - \beta_i w}.$$

Then, product l is more(less) elastic than product h if and only if

$$\begin{aligned}
\beta_l \frac{w}{\alpha_l - \beta_l w} &\gtreqless \beta_h \frac{w}{\alpha_h - \beta_h w} \\
&\Longrightarrow \delta = \frac{q_l - c_l}{q_h - c_h} \lesseqgtr 1,
\end{aligned}$$

which is exactly the same condition as given in the above proposition. This shows that the input monopolist's optimal pricing follows the standard inverse-elasticity rule.

All those results, taken together, indicates the following relations:

$$w_l^* \lesseqgtr w^* \lesseqgtr w_h^* \Longleftrightarrow \varepsilon_l \gtreqless \varepsilon_h \Longleftrightarrow \delta \lesseqgtr 1.$$

Note that $D_l^e - D_h^e = \frac{(q_h - c_h)(2q_h - q_l)}{2(4q_h - q_l)q_l} > 0$ for $\delta = 1$. So, the price elasticity is equalized for the two products when the input demand is larger for the low quality than for the high quality. This means that the price elasticity of demand for the low quality can be higher than the high quality even if its equilibrium demand is larger than the high-quality product. The low-quality firm faces competition not only from the high-quality firm but also from no purchasing option, and therefore its demand is more sensitive to a price change compared to that of the high-quality firm who, insulated from the (binding) consumer participation constraint, competes only with the low-quality firm product. Then, given $-\frac{dD_l}{dw} > -\frac{dD_h}{dw}$, in order for $\varepsilon_l \equiv -\frac{dD_l}{dw}\frac{w}{D_l} = -\frac{dD_h}{dw}\frac{w}{D_h} \equiv \varepsilon_h$ it must be that $D_l > D_h$. For $\delta > 1$, the effect of the participation constraint is dominated by the relative demand size of the low-quality product. This result contrasts with the one obtained in the Cournot model with homogeneous products such as Katz(1987) and DeGraba (1990). In the homogeneous goods model, all the firms face the same market demand, and therefore the relative elasticity of input demand depends solely on the size of equilibrium quantities (which in turn rely on the cost efficiency of the firms). We will see later how this difference plays a key role in driving our distinctive welfare results of input price discrimination.

Let us now compare other equilibrium properties of the two equilibria, focusing on the effect of input price discrimination on the final-product market. For linear demands, total output is the same in the two pricing regimes provided all markets are served under uniform pricing. This property applies to the present model as well, and immediately means that the price of the low-quality product does not change with price discrimination. Let p_i^u and p_i^d denote the price of quality-i product under uniform pricing and price discrimination, $i = l, h$. Let $\widetilde{\theta}_u$ and $\widetilde{\theta}_d$ denote the type of consumers who are indifferent between the two products under uniform pricing and price discrimination. Similarly, let $\widetilde{\widetilde{\theta}}_u$ and $\widetilde{\widetilde{\theta}}_d$ denote the indifferent type between the low quality and no purchase under uniform pricing and price discrimination.

Corollary 1 *i) $p_l^u = p_l^d$ and $\widetilde{\widetilde{\theta}}_u = \widetilde{\widetilde{\theta}}_d$ for all δ. ii) $p_h^u \lesseqqgtr p_h^d$ and $\widetilde{\theta}_u \lesseqqgtr \widetilde{\theta}_d$ if and only if $\delta \lesseqqgtr 1$.*

Part ii) is a direct consequence of Proposition 1. For $\delta < 1$, the price of the high-quality product increases under price discrimination with a higher input price, which obviously leads to a lower sales volume. The same thing happens for the low-quality product for $\delta > 1$. Also, the optimal input prices in Proposition 1 implies that the low-quality firm is better off while the high-quality firm is worse off under price discrimination for $\delta < 1$, and the opposite holds for $\delta > 1$.

2.3 Welfare effects of price discrimination

If price discrimination in *final*-goods markets is shown to increase consumer surplus, the overall welfare should also increase. Total welfare is the sum of consumer surplus and monopoly profits. Monopoly profits naturally increase under price discrimination. Therefore, if consumer surplus increases total welfare must increase as well. This proposition, however, does not always hold when price discrimination takes place in *intermediate*-goods markets. This is because firm profits consist of the sum of the input monopolist's and downstream firms' profits, and thus an increase in consumer surplus does not guarantee that total welfare will rise. If downstream firms' profits fall significantly, total welfare may decrease even if consumer surplus increases. This means that we need to consider the effects of input price discrimination on consumer surplus and total welfare separately. Furthermore, some competition authorities often use consumer surplus rather than total welfare as the welfare standard in antitrust enforcement (see Motta (2000) and Farrell and Katz (2006) for more detail on this issue).

Consumer surplus: Let us first examine the effect input price discrimination has on consumer surplus. The following result is immediate from Corollary 1.

Proposition 2 *Consumers are weakly worse(better) off under price discrimination for* $\delta < 1$*(*$\delta > 1$*).*

Since the price of the low-quality product remains the same in the two regimes, consumers who continue to buy the low-quality product are not affected by price discrimination. However, other consumers' surplus is affected by input price discrimination. For $\delta < 1$, price discrimination raises the price of the high-quality good ($p_h^u < p_h^d$), and all the other participating consumers become worse off due to this price change. Consumers of type in $[\widetilde{\theta}_u, \widetilde{\theta}_d]$, who would purchase the high-quality product under uniform pricing, are induced to switch to the low-quality product. Also, consumers of type in $[\widetilde{\theta}_d, 1]$ purchase the high-quality product at a higher price than under uniform pricing. Meanwhile, for $\delta > 1$ the price of the high-quality good decreases with price discrimination ($p_h^u > p_h^d$), and this makes all remaining consumers better off. In this case, consumers of type in $[\widetilde{\theta}_d, \widetilde{\theta}_u]$ enjoy a larger utility under price discrimination by purchasing the high-quality product, although they could buy the low-quality product at the same price. Consumers of type in $[\widetilde{\theta}_u, 1]$ become better off by consuming the high-quality product at a lower price.

The above result is new to the literature on input price discrimination. Most of previous works, based on a Cournot model in which consumer surplus is solely linked to the level of total output (which remains constant with a linear demand), failed to capture the diverse effects input price discrimination has on consumer surplus under product differentiation. Note that in our model consumer surplus changes with price discrimination even though total output remains constant under the two pricing regimes. Our result is also different from that found in the analysis of price discrimination in final-goods markets. As shown by Cowan (2012), monopoly third-degree price discrimination in a final-good market tends to harm consumers.[13] In contrast, we show that price discrimination can make all consumers

[13] Note that, however, price discrimination may increases aggregate consumer surplus if the ratio of pass-through to the price elasticity at the uniform price is the same or larger in the high-elasticity market.

(weakly) better off under a certain condition.

Policy guideline: Recall that the ratio of optimal input prices under price discrimination is equal to the cost-adjusted quality ratio, i.e. $\delta \equiv \frac{q_l - c_l}{q_h - c_h} = \frac{w_l^*}{w_h^*}$. Then, from the result in Proposition 2 we can easily see how consumers are affected by price discrimination by simply checking the ratio of input prices which are easily observed in the market: allow price discrimination if $w_l^* > w_h^*$ (i.e. $\delta > 1$) and ban it if $w_l^* < w_h^*$ (i.e. $\delta < 1$). This simple rule provides a useful guideline for policymakers adopting consumer surplus as a welfare standard and is expected to be widely used in real antitrust litigation.

Total welfare: Next we investigate how input price discrimination affects social welfare defined as the sum of consumer surplus and firm profits. Not surprisingly, the welfare effect depends on the level of the cost-adjusted quality ratio δ. What is interesting, however, is the fact that social welfare can rise with price discrimination even though it does not involve an increase in total output, which sharply contrasts with the result obtained in the previous literature.

Proposition 3 *There exists a cut-off value of δ, denoted $\delta^W \in (\bar{\delta}_h, 1)$, such that price discrimination increases social welfare for $\delta \in (\delta^W, 1)$ and reduces it for $\delta \in (\bar{\delta}_h, \delta^W) \cup (1, \bar{\delta}_l)$.*

As shown by Katz (1987) and DeGraba (1990), input price discrimination in a Cournot market with homogeneous goods tends to reduce social welfare. Under price discrimination a higher input price is charged for the firm with low marginal cost, who faces more elastic demand with a larger sales volume. It is like taxing an efficient firm, thus raising the total cost of production for a given output. This is not always true when firms compete with differentiated products and consumers are heterogeneous in their preferences for the products. As we have seen before, the low-quality firm, due to the consumer participation constraint, may face a lower input price even if it has a larger sales volume than the high-quality firm. This implies that the welfare result can be reversed (i.e. taxing a firm with less elastic input demand can improve welfare), given that the sales volume is positively correlated with the relative efficiency of the firms in terms of the cost-adjusted quality.

In order to see this, we first characterize the welfare-maximizing allocation of the two products. Let us define consumer type θ^s such that

$$\theta^s q_l - c_l = \theta^s q_h - c_h \Rightarrow \theta^s = \frac{c_h - c_l}{q_h - q_l}.$$

Then, it is socially desirable for consumers of type lower than θ^s to buy the low-quality product and for those of type higher than θ^s to buy the high-quality product. The following lemma exhibits the relationship between the welfare-maximizing customer segmentation and the cost-adjusted quality ratio.

Lemma 2 $\theta^s \lesseqqgtr 1$ *if and only if* $\delta \lesseqqgtr 1$.

Note that $w_l^* \lesseqqgtr w_h^*$ and $\theta^s \lesseqqgtr 1$ for $\delta \lesseqqgtr 1$. That is, when $\delta = 1$ it is welfare maximizing to allocate all consumers the low-quality product (except the highest type who is indifferent), and price discrimination has no impact on social welfare. This is an artifact of the normalization of consumer type to the unit interval.[14] Nevertheless, it provides a useful benchmark for our welfare analysis.

For $\delta > 1$ (i.e. $\theta^s > 1$), all types of consumers should be allocated the low-quality product for welfare maximization.[15] In this case, price discrimination always lowers social welfare by inducing consumers of type in $[\widetilde{\theta}_d, \widetilde{\theta}_u]$ to switch to the high-quality product from the more desirable low-quality product. For $\delta < 1$ (i.e. $\theta^s < 1$), however, welfare maximization involves consumers of type in $[\widetilde{\widetilde{\theta}}, \theta^s]$ buying the low-quality product and those of type in $[\theta^s, 1]$ buying the high-quality product. Then, price discrimination can increases social welfare if $\widetilde{\theta}_u < \theta^s$ and a sufficiently large fraction of consumers of type in $[\widetilde{\theta}_u, \widetilde{\theta}_d]$ switch to the more desirable low-quality product under price discrimination. In this case, the input monopolist charges a lower input price for the low-quality product even if its sales quantity is larger than the high-quality product (its demand is more elastic demand due to the consumer participation constraint). This reversion of optimal input prices with product differentiation is the main factor that enables the realization of welfare improvement.

The positive welfare result is most clearly demonstrated when $\delta = 1 - \epsilon$ (ϵ is small), i.e., the cost-adjusted quality is larger for the high-quality product but the gap between the two is very small. Note that $\widetilde{\theta}_u < \widetilde{\theta}_d < \theta^s = 1 - \frac{\epsilon(q_h - c_h)}{q_h - q_l}$ for $\delta = 1 - \epsilon$.[16] Price discrimination induces consumers of type in $[\widetilde{\theta}_u, \widetilde{\theta}_d]$ to buy the low-quality product instead of the high-quality product, and this consumption switching is always socially beneficial since $\widetilde{\theta}_u < \widetilde{\theta}_d < \theta^s$. Note that $\theta q_l - c_l > \theta q_h - c_h$ for all $\theta \in [\widetilde{\theta}_u, \widetilde{\theta}_d]$, given that $q_l - c_l < q_h - c_h$ and the gap between $q_l - c_l$ and $q_h - c_h$ is very small. As δ falls from 1, all three indifferent

[14] If the upper bound is greater than 1, θ^s can be an interior point of the type space. But it does not affect our welfare analysis.

[15] Note that $\theta q_l - c_l > \theta q_h - c_h$ for all $\theta \leq 1$, given that $\delta \equiv \frac{q_l - c_l}{q_h - c_h} > 1$.

[16] It can be shown that, for given q_h and q_l,

$$\theta^s - \tilde{\theta}_d = \frac{(q_h - c_h)[(7\delta - 6)\, q_h + q_l - 2\delta q_l]}{2\,(4q_h^2 - 5q_h q_l + q_l^2)} > 0$$

if and only if $\delta > \frac{6q_h - q_l}{7q_h - 2q_l}$. Note that $\frac{6q_h - q_l}{7q_h - 2q_l} < 1$ for $q_l < q_h$. Therefore, it must be that $\tilde{\theta}_u < \tilde{\theta}_d < \theta^s$ for $\delta = 1 - \epsilon$ (ϵ is small).

types decrease but θ^s decreases faster than the other two. Also, the gap between $\widetilde{\theta}_u$ and $\widetilde{\theta}_d$ gets larger as δ becomes smaller. At some point, it reaches to the point where $\widetilde{\theta}_u < \theta^s < \widetilde{\theta}_d$, so that switching of consumers of type $\theta \in [\widetilde{\theta}_u, \theta^s]$ is welfare improving while switching of those of type $\theta \in [\theta^s, \widetilde{\theta}_d]$ is detrimental to social welfare. As shown in Proposition 3, there exists a cut-off value of δ where the positive and negative impacts are precisely offset.

Perfect competition for each product: Suppose there is perfect or Bertrand competition for each of the two products. We assume $\delta < 1$ in order to focus on market sharing equilibria. Marginal cost pricing in the downstream markets yields the following equilibrium prices for given input prices w_l and w_h:

$$\begin{aligned} p_l^C &= c_l + w_l, \\ p_h^C &= c_h + w_h. \end{aligned}$$

Note that the indifferent type under uniform pricing ($w_l = w_h = w$) coincides with the indifferent type θ^s under welfare maximization (i.e. $\widetilde{\theta}_u = \frac{(c_h+w)-(c_h+w)}{q_h-q_l} = \frac{c_h-c_h}{q_h-q_l} = \theta^s$). Thus, price discrimination always lowers welfare as long as $\widetilde{\theta}_d \neq \widetilde{\theta}_u$. This result reveals that imperfect competition in the final-goods market is a necessary condition for input price discrimination to improve social welfare.

With imperfect competition, the downstream firms add margin on their effective marginal cost and so the prices of the final goods can be written as

$$\begin{aligned} p_l^I &= c_l + w_l + m_l, \\ p_h^I &= c_h + w_h + m_h, \end{aligned}$$

where m_i denotes the downstream margin put on the product of quality $i = l, h$. Now allocations under uniform pricing are inefficient whenever margins differ between the two products. That is, $\theta^s \neq \widetilde{\theta}_u$ if $m_l \neq m_h$ even if $w_l = w_h$. In fact, the following holds in our model:

$$\theta^s \gtreqless \widetilde{\theta}_u \text{ iff } m_l \gtreqless m_h.$$

For δ being not too small, the low-quality firm tends to put on a bigger margin since with a larger demand it is more sensitive the inframarginal effect of a price increase than the high-quality firm, although its marginal effect is also larger than the high quality firm due to the binding participation constraint.[17] This disequalization of margins leads to $\widetilde{\theta}_u < \theta^s$ in equilibrium. In particular $m_l > m_h$ so that $\widetilde{\theta}_u < \theta^s$ for $\delta = 1 - \epsilon$, in which case

[17] It can be shown that

$$m_l^u - m_h^u = \frac{(q_h - c_h)\left(2q_h^2 - q_h q_l - q_l^2\right)}{2\left(4q_h - q_l\right)\left(2q_h + q_l\right)} > 0$$

for $\delta = 1$ under uniform pricing.

price discrimination increases total welfare by inducing consumers of type in $[\widetilde{\theta}_u, \widetilde{\theta}_d]$ to switch the more socially desirable low-quality product. As δ falls, the difference in margin gets smaller and the positive welfare effect eventually disappears and the welfare effect is reversed at some point.

Policy guideline: As before, the above result provides some guidance for policymakers who use total welfare as a welfare standard. We know that price discrimination enhances total welfare if $\widetilde{\theta}_u < \widetilde{\theta}_d < \theta^s$ for $\delta < 1$. Note that $\widetilde{\theta}_u < \widetilde{\theta}_d$ if $\delta = \frac{w_l^*}{w_h^*} < 1$. Also note that

$$\begin{aligned}
\widetilde{\theta}_d &= \frac{(c_h + w_h^* + m_h) - (c_l + w_l^* + m_l)}{q_h - q_l} < \theta^s = \frac{c_h - c_l}{q_h - q_l} \\
&\Leftrightarrow (w_h^* + m_h) - (w_l^* + m_l) < 0.
\end{aligned}$$

A sufficient condition for price discrimination to increase social welfare is that the input price is higher for the high quality while the sum of the input price and downstream margin is higher for the low quality under price discrimination, i.e. $w_l^* < w_h^*$ and $w_l^* + m_l > w_h^* + m_h$. So the welfare effect can be easily assessed by observing or estimating downstream margins. Since $p_i^e = c_i + w_i^* + m_i$, the margins can be inferred from price data p_i^e and w_i^* if we know downstream marginal costs.

Policy implication: Our welfare analysis shows that price discrimination has different effects on consumer surplus and total welfare. When $\delta < 1$, the discriminatory input monopolist charges a higher input price to the high quality and a lower price to the low quality. This obviously makes the high-quality firm worse off and the low-quality firm better off. All consumers are weakly worse off with a price increase of the high-quality product (the price of the low-quality product remains the same). Price discrimination induces some consumers to switch to the low quality from the high quality, which increases social welfare for $\delta > \delta^W$ and decreases it for $\delta < \delta^W$. When $\delta > 1$, price discrimination raises the input price for the low quality and lowers the input price for the high quality. So, the high-quality firm is better off while the low-quality firm is worse off under price discrimination. Consumers become weakly better off because the price of the high-quality product falls and the price of the low-quality product remains the same. However, price discrimination reduces total welfare by inducing some consumers to switch inefficiently to the high-quality product. Since the input monopolist certainly gains from price discrimination, this implies that the profit loss for the low-quality firm outweighs the sum of the gains of the other parties. The following table summarize the welfare effects of price discrimination according to the value of the cost-adjusted quality ratio δ. Different policy responses are required depending on the welfare standard.

Table 1: Welfare effects of price discrimination and policy responses

	CS	SW	Policy Response
$\delta < \delta^W$	$\downarrow$	$\downarrow$	ban regardless of welfare standard
$\delta^W < \delta < 1$	$\downarrow$	$\uparrow$	ban only under consumer surplus standard
$\delta > 1$	$\uparrow$	$\downarrow$	ban only under total welfare standard

The following figure shows how welfare effects of price discrimination differ according to welfare standards as a function δ for the parameter values of $q_h = 2, q_l = 1, c_l = 0$.

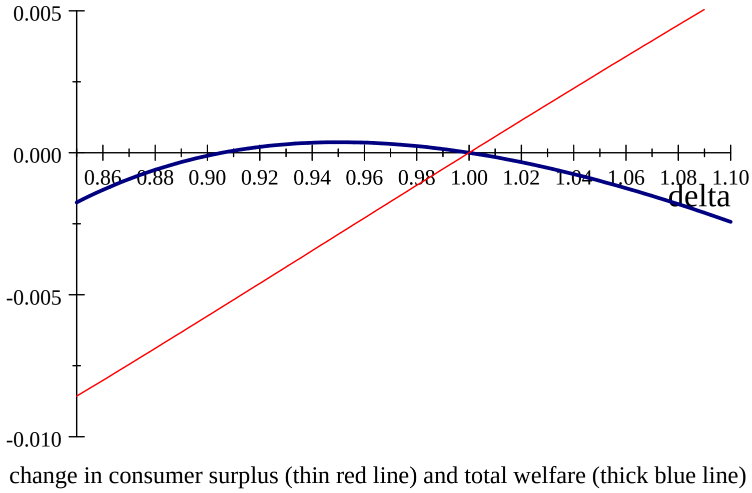

Figure 1. Welfare effects of input price discrimination

3 Horizontal differentiation

Now we extend analysis to the case of horizontal differentiation using the Hotelling linear-city model. Our objective is to show that the results obtained in the previous vertical differentiation model continues to hold with horizontally differentiated products.

Consider an upstream monopolist selling an input to two downstream firms, denoted A and B, producing horizontally differentiated products which give value v to all consumers. Suppose firm A is located at $l_A \in (0, 1)$ and firm B at $l_B = 1$. This asymmetric location setup is similar to vertical differentiation and plays a crucial role in driving our welfare result. There is a continuum of consumers of mass 1 with unit demand, and consumer type x is uniformly distributed on the unit interval $[0, 1]$. Let $u(x) = v - t\,|x - l_i| - p_i$ be the utility a consumer of type x obtains when purchasing a unit of product located at l_i at price p_i, where t is the transportation cost per unit of distance. The rest of the model specification is the same as before. The input is used in a fixed proportion (one-to-one) to produce a unit of the final goods. The input monopolist produces at zero marginal cost, and the downstream firms at constant marginal cost $c_i, i = A, B$, in addition to the cost of inputs. The upstream monopolist quotes input price w_i to the downstream firm located at l_i and then the downstream firms simultaneously choose retail price p_i for the final goods.

The price equilibrium of the final goods given input prices w_A and w_B is characterized as follows. Given p_A and p_B, two indifferent types are defined as follows:

$$v - t\left(\widetilde{x} - l_A\right) - p_A = v - t\left(1 - \widetilde{x}\right) - p_B \Rightarrow \widetilde{x} = \frac{1}{2} + \frac{p_B - p_A}{2t} + \frac{l_A}{2} \tag{4}$$

and

$$v - t(l_A - \widetilde{\widetilde{x}}) - p_A = 0 \Rightarrow \widetilde{\widetilde{x}} = l_A - \frac{v - p_A}{t}, \tag{5}$$

where the consumers of type $x \in [\widetilde{\widetilde{x}}, \widetilde{x}]$ buy the product of firm A, and those of type $x \in [\widetilde{x}, 1]$ buy the product of firm B. It is obvious that $\widetilde{\widetilde{x}} < l_A < \widetilde{x} < l_B = 1$. We confine our analysis to the case where l_A is far away from 0 and v is sufficiently small and t is large so that the consumers located near to $x = 0$ do not purchase (i.e. $\widetilde{\widetilde{x}} > 0$) and the utility of type-$\widetilde{x}$ consumers is nonnegative in equilibrium.[18] Specific conditions for this to happen are provided below.

Then, the demands for the two products can be written as

$$D_A(p_A, p_B) = \widetilde{\widetilde{x}} - \widetilde{x} = \frac{v}{t} + \frac{1 - l_A}{2} - \frac{3p_A - p_B}{2t}$$

and

$$D_B(p_A, p_B) = 1 - \widetilde{x} = \frac{1}{2} - \frac{p_B - p_A}{2t} - \frac{l_A}{2}.$$

The firm A's profit-maximization problem is

$$\max_{p_A} : \left(p_A - w_A - c_A\right)\left(\frac{v}{t} + \frac{1 - l_A}{2} - \frac{3p_A - p_B}{2t}\right),$$

which gives the reaction function:

$$p_A^R(p_B) = \frac{1}{6}\left[p_A + 2v + t\left(1 - l_A\right) + 3\left(w_A + c_A\right)\right].$$

[18] Price discrimination always reduces welfare if the consumers located on the two extremities in [0,1] obtain strictly positive net utility in equilibrium (i.e. $\widetilde{\widetilde{x}} < 0$). This is because in this case the relative elasticity of input demand for the two products depends solely on the sales volumes, as in the case of homogeneous goods. This implies that for the positive welfare result it is required that the demands of the two downstream firms should be asymmetric in term of who is facing the binding consumer participation constraint.

Similarly, from the firm B's profit-maximization problem,

$$\max_{p_B} : (p_B - w_B - c_B) \left(\frac{1}{2} - \frac{p_B - p_A}{2t} - \frac{l_A}{2} \right),$$

the following reaction function is obtained:

$$p_B^R(p_A) = \frac{1}{2} \left[p_A + t(1 - l_A) + (w_B + c_B) \right].$$

Solving the reaction functions yields the following equilibrium prices and demands of the two products:

$$\begin{aligned} p_A^e &= \tfrac{1}{11} \left[4v + 3t(1 - l_A) + 6(w_A + c_A) + (w_B + c_B) \right], \\ p_B^e &= \tfrac{1}{11} \left[2v + 7t(1 - l_A) + 3(w_A + c_A) + 6(w_B + c_B) \right]. \end{aligned} \tag{6}$$

$$\begin{aligned} D_A^e &= \tfrac{3}{22t} \left[4v + 3t(1 - l_A) - 5(w_A + c_B) + (w_B + c_B) \right], \\ D_B^e &= \tfrac{1}{22t} \left[2v + 7t(1 - l_A) + 3(w_A + c_B) - 5(w_B + c_B) \right]. \end{aligned} \tag{7}$$

3.1 Optimal input prices

Assuming the input monopolist can use only linear pricing, we can derive the optimal input prices under price discrimination and uniform pricing as follows.

Price discrimination: The input monopolist chooses w_A and w_B to maximize $w_A D_A^e + w_B D_B^e$. Assuming a positive sales volume for both products, the optimal input prices are given as

$$w_A^* = \frac{1}{2} \left[v + t(1 - l_A) - c_A \right], w_B^* = \frac{1}{2} \left[v + 2t(1 - l_A) - c_B \right], \tag{8}$$

which are valid only if $0 < \tilde{\tilde{x}} < l_A < \tilde{x} < l_B = 1$. For similar marginal costs, a lower input price is charged for product A even though its demand is greater than the rival product B. This is because the demand firm A facing is more elastic due to the consumer participation constraint on the left side of the unit interval.

As before, we can define the following ratio index:

$$\delta' \equiv \frac{v + t(1 - l_A) - c_A}{v + 2t(1 - l_A) - c_B} = \frac{w_A^*}{w_B^*}, \tag{9}$$

which indicates the relative competitive advantage of the two products in the market. Product A located at l_A becomes more advantageous as δ' increases. From (8) and (9) we can derive the following equation:

$$c_A = v + t(1 - l_A) - \delta' \left[v + 2t(1 - l_A) - c_B \right]. \tag{10}$$

Plugging in the optimal input prices in (8) and substituting c_A in (10), the conditions $0 < \widetilde{\widetilde{x}} < l_0 < \widetilde{x} < l_1$ and $u(\widetilde{x}) > 0$ are reduced to

$$\max\left\{-\tfrac{1}{6} + \tfrac{11t(1-l_A)}{3[v+2t(1-l_A)-c_B]}, \tfrac{5}{3} - \tfrac{44t(1-l_A)}{[v+2t(1-l_A)-c_B]}\right\} < \delta' \\ < \min\left\{\tfrac{5}{3}, -\tfrac{1}{6} + \tfrac{11t}{3[v+2t(1-l_A)-c_B]}, -\tfrac{7}{9} + \tfrac{88t(1-l_A)}{9[v+2(1-l_A)t-c_B]}\right\}$$

Uniform pricing: The input monopolist chooses w to maximize $w(D_0^e + D_1^e)$. Assuming a positive sales volume for both products, the optimal uniform price is given by

$$w^* = \frac{1}{2}v + \frac{1}{14}\left[8t\left(1 - l_A\right) - 6c_A - c_B\right], \tag{11}$$

which is valid only if $0 < \widetilde{\widetilde{x}} < l_0 < \widetilde{x} < 1$ and $u(\widetilde{x}) > 0$. These conditions, after substituting the optimal input price in (11), reduce to

$$\max\left\{-\tfrac{1}{6} + \tfrac{11t(1-l_A)}{3[v+2t(1-l_A)-c_B]}, \tfrac{34}{27} - \tfrac{154t(1-l_A)}{27[v+2t(1-l_A)-c_B]}\right\} < \delta' \\ < \min\left\{\tfrac{34}{27}, -\tfrac{1}{6} + \tfrac{11t}{3[v+2t(1-l_A)-c_B]}, -\tfrac{41}{15} + \tfrac{308t(1-l_A)}{15[v+2t(1-l_A)-c_B]}\right\}$$

The joint satisfaction of $0 < \widetilde{\widetilde{x}} < l_0 < \widetilde{x} < 1$ and $u(\widetilde{x}) > 0$ in the two regimes requires that

$$\underline{\delta} < \delta' < \overline{\delta}, \tag{12}$$

where $\underline{\delta} \equiv \max\left\{-\tfrac{1}{6} + \tfrac{11t(1-l_A)}{3[v+2t(1-l_A)-c_B]}, \tfrac{34}{27} - \tfrac{154t(1-l_A)}{27[v+2t(1-l_A)-c_B]}, \tfrac{5}{3} - \tfrac{44t(1-l_A)}{[v+2t(1-l_A)-c_B]}\right\}$ and $\overline{\delta} \equiv \min\left\{\tfrac{34}{27}, -\tfrac{1}{6} + \tfrac{11t}{3[v+2t(1-l_A)-c_B]}, -\tfrac{7}{9} + \tfrac{88t(1-l_A)}{9[v+2(1-l_A)t-c_B]}, -\tfrac{41}{15} + \tfrac{308t(1-l_A)}{15[v+2t(1-l_A)-c_B]}\right\}$. We will assume that condition (12) is satisfied in the following analysis, i.e. l_A is sufficiently away from 0.[19]

3.2 Uniform pricing vs price discrimination

Comparing the optimal input prices in (8) and (11) leads to the following proposition. Similar to the case of vertical differentiation, the optimal input pricing is closely related to the price elasticities of (linear) input demands of the two products.

Proposition 4 *$w_0^* \lesseqqgtr w^* \lesseqqgtr w_1^*$ for $\delta' \lesseqqgtr 1$.*

[19] The parameter space satisfying condition (12) is non-empty. For example, for $v = 5$, $t = 3$, $l_A = 0.5$ and $c_A = 1$ the low and upper bounds are $\underline{\delta} \simeq 0.6190$ and $\overline{\delta} \simeq 1.3175$ respectively.

With linear demands, total output is identical under the two pricing regimes. So the price of the product A does not change with price discrimination. Let p_i^u and p_i^d denote equilibrium price of product i under uniform pricing and price discrimination, $i = A, B$. Let $\widetilde{x}_u$ and $\widetilde{x}_d$ denote the indifferent type between the two products under uniform and discriminatory pricing. Similarly, let $\widetilde{\widetilde{x}}_u$ and $\widetilde{\widetilde{x}}_d$ denote the indifferent type between the product A and no purchase under uniform and discriminatory pricing. The following corollary summarizes the effects of input price discrimination on the prices and sales quantities of the final products, which are analogous to those obtained in Corollary 1 for the case of vertical differentiation.

Corollary 2 *i)* $p_A^u = p_A^d$ *and* $\widetilde{\widetilde{x}}_u = \widetilde{\widetilde{x}}_d$ *for all* δ'. *ii)* $p_B^u \lesseqgtr p_B^d$ *and* $\widetilde{x}_u \lesseqgtr \widetilde{x}_d$ *if and only if* $\delta' \lesseqgtr 1$.

Input price discrimination raises the price of the product B for $\delta' < 1$ and the price of the product A for $\delta' > 1$. So, it makes firm A better off and firm B worse off for $\delta' < 1$, and conversely for $\delta' > 1$.

3.3 Welfare effects of price discrimination

The following result, which is immediate from Corollary 2, summarizes the effect of price discrimination on consumer welfare.

Proposition 5 *Consumers are weakly worse(better) off under price discrimination for* $\delta' < 1$*(*$\delta' > 1$*).*

The intuition is basically identical to the case of vertical differentiation. Given the same price for product A in both regimes, consumers who continue to purchase product A under price discrimination are not affected by price discrimination. For $\delta' < 1$, consumers who switch to product A or continue to purchase product B become worse off under price discrimination. On the other hand, for $\delta' > 1$, consumers who switch to product B or continue to purchase product B become better off under price discrimination.

As before, input price discrimination can increase social welfare by inducing some consumers to switch to a socially more desirable product.

Proposition 6 *Price discrimination increases total welfare when* δ' *is less than but close to 1.*

Instead of characterizing the region of δ' in which price discrimination is welfare improving, we content ourselves with showing that welfare improvement is possible for $\delta' < 1$.

Let us define consumer type x^s such that

$$v - t(x^s - l_A) - c_A = v - t(1 - x^s) - c_B$$
$$\Rightarrow x^s = \frac{1 + l_A}{2} - \frac{c_A - c_B}{2t}. \tag{13}$$

Plugging in (10) into (13), we see that the following relation holds:

$$x^s \lesseqgtr 1 \text{ if and only if } \delta' \lesseqgtr 1.$$

For $\delta' > 1$ (i.e. $x^s > 1$), it is welfare maximizing to allocate all consumers product A. In this case, price discrimination always reduces social welfare by inducing consumers of type in $[\widetilde{x}_d, \widetilde{x}_u]$ to switch to the socially less desirable product B. For $\delta' < 1$ (i.e. $x^s < 1$), however, it is welfare maximizing to allocate product A to consumers of type lower than x^s and product B to those of type higher than x^s. Then, price discrimination can increase welfare if $\widetilde{x}_u < x^s$ and a sufficiently large fraction of consumers of type in $[\widetilde{x}_u, \widetilde{x}_d]$ switch from product B to product A with price discrimination. For instance, it can be easily seen that this consumption switching is socially desirable if $\widetilde{x}_u < \widetilde{x}_d \leq x^s$. We know that $\widetilde{x}_u < \widetilde{x}_d$ for $\delta' < 1$. From (4) and (8), the following relation holds:

$$\begin{aligned} \widetilde{x}_d \quad &= \quad \frac{3c_B - c_A + t(43 + l_A) - 2v}{44t} \leqq x^s = \frac{1 + l_A}{2} - \frac{c_A - c_B}{2t} \\ \iff \quad & \delta' \geq \frac{23}{25}, \end{aligned}$$

where the use was made of (10). So in this case price discrimination definitely increases social welfare if $\frac{23}{25} < \delta' < 1$. For $\delta' < \frac{23}{25}$, however, it holds that $x^s < \widetilde{x}_d$. Therefore, consumers of type in $[x^s, \widetilde{x}_d]$ are induced to switch from product B to product A under price discrimination, which is detrimental to social welfare. On the other hand, price discrimination improves welfare by inducing consumers of type in $[\widetilde{x}_u, x^s]$ to switch from product A to product B. If δ' is sufficiently small the negative effect will dominate the positive effect and therefore there exists a cut-off value of δ' in $(0, \frac{23}{25})$ where the welfare effect is reversed.

4 Conclusion and discussion

We extended the analysis on input price discrimination to an environment where downstream firms compete with spatially differentiated products for which consumers have heterogeneous preferences. The main findings of the analysis are as follows.

First, the input monopolist's optimal pricing follows the standard inverse-elasticity rule and is closely related to the cost-adjusted quality(location) ratio of the final products.

Second, input price discrimination can raise social welfare even though it does not expand total output. So we cannot assert that price discrimination is socially harmful just because it does not increase total sales quantity. Third, the effect of input price discrimination on consumer surplus is quite different from its effect on total welfare and, therefore, different policy responses are required depending on welfare standard. Fourth, we proposed simple policy tools that can be used in real antitrust cases: the ratio of optimal input prices is useful for evaluating the effect of price discrimination on consumer surplus, and marginal costs or profit margins of downstream firms can be used to assess the effect on total welfare.

We analyzed a stylized model with linear demands for two differentiated products. Our predictions would go through even if we allow for nonlinear demands as long as those are constructed through a small continuous perturbation of the linearity. Moreover, nonlinear demands might extend the positive welfare effect of price discrimination even further, that is, price discrimination may increase social welfare even if it strictly decreases total output. It will be interesting to see what would happen if the input monopolist can use two-part tariffs. The availability of two-part tariff contracts dramatically change the optimal input pricing and its welfare effect, as shown by Inderst and Shaffer (2009) in the context of representative consumer demands. Discriminatory two-part tariffs increase allocative efficiency by favoring the more productive firm and thereby raise consumer and social surplus. A similar reversal is expected in our model of product differentiation as well. With two-part tariffs, the input monopolist would lower the per-unit wholesale price for the relatively efficient firm under price discrimination, which tends to increase social welfare. With product differentiation, however, price discrimination may lead to a decrease in welfare by inducing some consumers to inefficiently change consumption choices. In particular, for the cost-adjusted quality ratio being smaller than and close to 1 some consumers will inefficiently switch from the high-quality good to the low-quality good under discriminatory two-part tariffs and this negative welfare effect may outweigh the positive welfare effect of efficient production and output expansion. This sharply contrasts with the result obtained in our analysis under linear wholesale prices and also with the one of Inderst and Shaffer (2009). Also, it will be worthwhile to endogenize product quality or brand location and see how it affects the optimal pricing rule and the welfare results.

5 Appendix

Proof of lemma 1

i) $\delta_h \equiv \dfrac{q_l}{2q_h - q_l} = \dfrac{\delta_P}{2-\delta_P} < \delta_P$, given that $\delta_P \equiv \dfrac{q_l}{q_h} < 1$.

ii) $\delta_P < \overline{\delta}_h \equiv \dfrac{3q_h q_l}{2q_h^2 + 2q_h q_l - q_l^2} = \dfrac{3\delta_P}{2 + 2\delta_P - \delta_P^2}$, given that $\delta_P^2 - 2\delta_P + 1 > 0$.

iii) $\overline{\delta}_h \equiv \dfrac{3\delta_P}{2 + 2\delta_P - \delta_P^2} < 1$ since $\delta_P^2 + \delta_P - 2 < 0$ for $\delta_P \in (0.1)$.

iv) $1 < \overline{\delta}_l \equiv \dfrac{8q_h^2 - q_h q_l - q_l^2}{6q_h^2} = \dfrac{8 - \delta_P - \delta_P^2}{6}$ since $2 - \delta_P - \delta_P^2 > 0$ for $\delta_P \in (0.1)$.

v) $\overline{\delta}_l \equiv \dfrac{8q_h^2 - q_h q_l - q_l^2}{6q_h^2} = \dfrac{8 - \delta_P - \delta_P^2}{6} < 2 - \delta_P = \dfrac{2q_h - q_l}{q_h} \equiv \delta_l$ for $\delta_P \in (0.1)$.

Proof of proposition 1

For $\delta = \frac{q_l - c_l}{q_h - c_h} \leqq 1$ (i.e. $q_l - c_l \leqq q_h - c_h$), it holds that

$$w_l^* = \frac{q_l - c_l}{2} \leqq \frac{q_l(q_l - c_l) + 2q_h(q_l - c_l)}{2(q_l + 2q_h)} \leqq \frac{q_l(q_h - c_h) + 2q_h(q_l - c_l)}{2(q_l + 2q_h)} = w^*$$

and

$$w^* = \frac{q_l(q_h - c_h) + 2q_h(q_l - c_l)}{2(q_l + 2q_h)} \leqq \frac{q_l(q_h - c_h) + 2q_h(q_h - c_h)}{2(q_l + 2q_h)} \leqq \frac{q_h - c_h}{2} = w_h^*,$$

which lead to $w_l^* \leqq w^* \leqq w_h^*$. Similarly, for $\delta = \frac{q_l - c_l}{q_h - c_h} \geqq 1$ (i.e. $q_l - c_l \geqq q_h - c_h$) it can be easily verified that $w_l^* \geqq w^* \geqq w_h^*$.

Proof of corollary 1

i) Total demand is given by

$$1 - \widetilde{\widetilde{\theta}} \equiv 1 - \frac{p_l^e}{q_l} = 1 - \frac{1}{q_l}\frac{q_l(q_h - q_l) + 2q_h(w_l^e + c_l) + q_l(w_h^e + c_h)}{4q_h - q_l},$$

which is identical under the two regimes if and only if

$$2q_h w_l^* + q_l w_h^* = (2q_h + q_l)w^*.$$

Plugging in the equilibrium input prices in (2) and (3), we can verify that

$$\begin{aligned} 2q_h w_l^* + q_l w_h^* &= 2q_h \frac{q_l - c_l}{2} + q_l \frac{q_h - c_h}{2} \\ &= \frac{q_l(q_h - c_h) + 2q_h(q_l - c_l)}{2} = (2q_h + q_l)w^*, \end{aligned}$$

thus total demand and, therefore, the price of the low-quality product are indeed the same under the two pricing regimes.

ii) Note that

$$\widetilde{\theta}_u - \widetilde{\theta}_d = \frac{(p_h^u - p_h^d) - (p_l^u - p_l^d)}{q_h - q_l}.$$

Since $p_l^u = p_l^d$ given that $\widetilde{\widetilde{\theta}}_u = \widetilde{\widetilde{\theta}}_d$, the following must hold:

$$sign(\widetilde{\theta}_u - \widetilde{\theta}_d) = sign\left(p_h^u - p_h^d\right).$$

Substituting the equilibrium prices in (2) and (3), we obtain

$$\begin{aligned} p_h^u - p_h^d &= 2q_h(w^* - w_h^*) + q_h(w^* - w_l^*) \\ &= \frac{q_h\,(4q_h - q_l)}{2(2q_h + q_l)}\,(c_h - q_h - c_l + q_l) \\ &= \frac{q_h\,(4q_h - q_l)}{2(2q_h + q_l)}\,(c_h - q_h - c_l + q_l)\,, \end{aligned}$$

which implies that

$$sign(p_h^u - p_h^d) = sign(c_h - q_h - c_l + q_l).$$

So, the following relation holds:

$$\begin{aligned} &\widetilde{\theta}_u \gtreqless \widetilde{\theta}_d \text{ and } p_h^u \gtreqless p_h^d \\ &\iff c_h - q_h - c_l + q_l \gtreqless 0 \\ &\iff \frac{q_l - c_l}{q_h - c_h} \equiv \delta \gtreqless 1. \end{aligned}$$

Proof of proposition 3

Let us denote the type of consumers who are indifferent between purchasing the high-quality and low-quality product under uniform pricing and price discrimination $\widetilde{\theta}_u$ and $\widetilde{\theta}_d$, respectively. As shown above, total demand is the same in the the two regimes (i.e., $\widetilde{\widetilde{\theta}}_u = \widetilde{\widetilde{\theta}}_d = \widetilde{\widetilde{\theta}}$). Let us denote the social welfare under uniform pricing and price discrimination as W^u and W^d respectively. The welfare difference between the two regimes is given by

$$\begin{aligned} W^d - W^u &= \left[\int_{\widetilde{\theta}_d}^1 \theta q_h d\theta - c_h(1 - \widetilde{\theta}_d) + \int_{\widetilde{\widetilde{\theta}}}^{\widetilde{\theta}_d} \theta q_l d\theta - c_l(\widetilde{\theta}_d - \widetilde{\widetilde{\theta}})\right] \\ &\quad - \left[\int_{\widetilde{\theta}_u}^1 \theta q_h d\theta - c_h(1 - \widetilde{\theta}_u) + \int_{\widetilde{\widetilde{\theta}}}^{\widetilde{\theta}_u} \theta q_l d\theta - c_l(\widetilde{\theta}_u - \widetilde{\widetilde{\theta}})\right] \\ &= \left(\widetilde{\theta}_u - \widetilde{\theta}_d\right)\left[\frac{1}{2}(q_h - q_l)\left(\widetilde{\theta}_u + \widetilde{\theta}_d\right) - (c_h - c_l)\right]. \end{aligned}$$

First, note that $\widetilde{\theta}_u - \widetilde{\theta}_d \gtreqless 0$ if and only if $\delta \gtreqless 1$ from corollary 1. Next, consider the sign of the second bracketed term. Plugging in the equilibrium values and substituting $c_l = q_l - \delta(q_h - c_h)$, we can rearrange the second bracketed term as

$$\begin{aligned}
& \frac{1}{2}(q_h - q_l)\left(\widetilde{\theta}_u + \widetilde{\theta}_d\right) - (c_h - c_l) \\
= \quad & \frac{(q_h - c_h)}{32q_h^2 - 4q_l^2 + 8q_h q_l}\left[4(5-6\delta)q_h^2 + (9-7\delta)q_h q_l + 2(2\delta - 1)q_l^2\right].
\end{aligned}$$

So, the following relation must hold:

$$\begin{aligned}
\frac{1}{2}(q_h - q_l)\left(\widetilde{\theta}_u + \widetilde{\theta}_d\right) - (c_h - c_l) \quad & \gtreqless \quad 0 \\
& \iff \quad 4(5-6\delta)q_h^2 + (9-7\delta)q_h q_l + 2(2\delta-1)q_l^2 \gtreqless 0 \\
& \iff \quad \delta \lesseqgtr \frac{20q_h^2 + 9q_h q_l - 2q_l^2}{24q_h^2 + 7q_h q_l - 4q_l^2} \equiv \frac{20 + 9\delta_P - 2\delta_P^2}{24 + 7\delta_P - 4\delta_P^2} \equiv \delta^W,
\end{aligned}$$

where we have substituted $\delta_P \equiv \frac{q_l}{q_h}$ in the last equation.

Then, the sign of the welfare difference after all depends on two conditions $\delta \gtreqless 1$ and $\delta \lesseqgtr \delta^W$. The following facts are useful for the proof.

i) $\delta^W < 1$:

$$\begin{aligned}
\delta^W \quad & \equiv \quad \frac{20 + 9\delta_P - 2\delta_P^2}{24 + 7\delta_P - 4\delta_P^2} < 1 \\
& \iff \quad 2\delta_P^2 + 2\delta_P - 4 < 0,
\end{aligned}$$

which always holds for $\delta_P \in [0, 1]$.

ii) $\bar{\delta}_h < \delta^W$:

$$\begin{aligned}
\bar{\delta}_h \quad & \equiv \quad \frac{3\delta_P}{2 + 2\delta_P - \delta_P^2} < \frac{20 + 9\delta_P - 2\delta_P^2}{24 + 7\delta_P - 4\delta_P^2} \equiv \delta^W \\
& \iff \quad 0 < (4 - \delta_P)(1 - \delta_P)(2 + \delta_P)(5 + 2\delta_P),
\end{aligned}$$

which always holds for $\delta_P \in [0, 1]$.

iii) $1 < \bar{\delta}_l$: Obvious from lemma 1 above.

Then, it is immediate that $W^d - W^u > 0$ for $\delta^W < \delta < 1$, and $W^d - W^u < 0$ for either $\bar{\delta}_h < \delta < \delta^W$ or $1 < \delta < \bar{\delta}_l$.

Proof of proposition 4

It always holds that $\delta' \lesseqgtr 1 \iff w_A^* \lesseqgtr w_B^*$ by definition. Also note that

$$\frac{6}{7}w_A^* + \frac{1}{7}w_B^* = \frac{1}{2}v + \frac{1}{14}\left[8t(1 - l_A) - 6c_A - c_B\right] = w^*,$$

which means that w^* is a weighted sum of w_A^* and w_B^*. Therefore, it must be that $w_A^* \lesseqgtr w^* \lesseqgtr w_B^*$.

Proof of corollary 2

From (6) it follows that

$$\begin{aligned} p_A^d &= \frac{1}{11}\left[4v + 3t\left(1 - l_A\right) + 6\left(w_A^* + c_A\right) + \left(w_B^* + c_B\right)\right] \\ p_A^u &= \frac{1}{11}\left[4v + 3t\left(1 - l_A\right) + 6\left(w^* + c_A\right) + \left(w^* + c_B\right)\right]. \end{aligned}$$

So, $p_A^u = p_A^d$ if and only if $6w_A^* + w_B^* = 7w^*$, which has been proved in Proposition 4. Since $\widetilde{\widetilde{x}}_j = l_A - \frac{v-p_A^j}{t}$, $j = u, d$ from (5), it follows that $\widetilde{\widetilde{x}}_u = \widetilde{\widetilde{x}}_d$ if $p_A^u = p_A^d$.

This result along with (4) implies that

$$\widetilde{x}_u - \widetilde{x}_d = \frac{p_B^u - p_A^u}{2t} - \frac{p_B^d - p_A^d}{2t} = \frac{p_B^u - p_B^d}{2t},$$

which shows that $\widetilde{x}_u \lesseqgtr \widetilde{x}_d$ if and only if $p_B^u \lesseqgtr p_B^d$. Now we prove that $p_B^u \lesseqgtr p_B^d$ if and only if $\delta' \lesseqgtr 1$. Substituting the equilibrium prices in (6) and using the fact that $6w_A^* + w_B^* = 7w^*$, we obtain

$$\begin{aligned} p_B^u - p_B^d &= \frac{1}{11}\left(9w^* - 3w_A^* - 6w_B^*\right) \\ &= \frac{1}{11}\left(9w^* - 3w_A^* - 6(7w^* - 6w_A^*)\right) \\ &= -3\left(w^* - w_A^*\right) \end{aligned}$$

Recall that $w_A^* \lesseqgtr w^* \lesseqgtr w_B^*$ if and only if $\delta' \lesseqgtr 1$. Thus, the following relation must hold:

$$\begin{aligned} &\widetilde{x}_u \lesseqgtr \widetilde{x}_d \text{ and } p_B^u \lesseqgtr p_B^d \\ &\iff w^* - w_A^* \gtreqless 0 \\ &\iff \delta' \gtreqless 1. \end{aligned}$$

References

[1] Aguirre, I., Cowan, S. and Vickers, J. (2010), Monopoly price discrimination and demand curvature, American Economic Review, 100, 1601–1615.

[2] Arya, A. and Mittendorf, B. (2010), Input price discrimination when buyers operate in multiple markets, Journal of Industrial Economics, 58, 846-867.

[3] Carlton, D. and Shampine, A. (2013), An economic interpretation of FRAND, Journal of Competition Law & Economics, 9, 531–552.

[4] Cowan, S. (2007), The welfare effects of third-degree price discrimination with non-linear demand functions, RAND Journal of Economics 32, 419-428.

[5] Cowan, S. (2012), Third-degree price discrimination and consumer surplus, Journal of Industrial Economics, 60, 333-345.

[6] Cowan, S. (2016), Welfare-increasing third-degree price discrimination, RAND Journal of Economics, 47, 326-340.

[7] Cowan, S. (2017), A model of third-degree price discrimination with positive welfare effects, Working paper.

[8] DeGraba, P. (1990), Input market price discrimination and the choice of technology, American Economic Review, 80, 1246–1253

[9] Dertwinkel-Kalt, M., Haucap, J. and Wey, C. (2016), Procompetitive dual pricing, European Journal of Law and Economics, 41, 537-557

[10] Farrell, J. and Katz, M. (2006), The economics of welfare standards in antitrust, Competition Policy International, 2(2), 2–28.

[11] Hausman, J. A. and MacKie-Mason, J. (1988), Price discrimination and patent policy, RAND Journal of Economics, 19, 253-265.

[12] Herweg, F. and Müller, D. (2012), Price discrimination in input markets: Downstream entry and efficiency, Journal of Economics and Management Strategy, 21, 773–799

[13] Herweg, F. and Müller, D. (2014), Price discrimination in input markets: Quantity discounts and private information, Economic Journal, 124, 776–804.

[14] Herweg, F. and Müller, D. (2016), Discriminatory nonlinear pricing, fixed costs, and welfare in intermediate-goods markets, International Journal of Industrial Organization, 46, 107–136.

[15] Holmes, T. (1989), The effects of third-degree price discrimination in oligopoly, American Economic Review, 79, 244-250.

[16] Inderst, R. and Shaffer, G. (2009), Market power, price discrimination, and allocative efficiency in intermediate-good markets, RAND Journal of Economics, 40, 658–672.

[17] Inderst, R. and Valletti, T. (2009), Price discrimination in input markets, RAND Journal of Economics, 40, 1–19.

[18] Kaftal, V. and Pal, D. (2008), Third degree price discrimination in linear-demand markets: effects on number of markets served and social welfare, Southern Economic Journal, 75, 558-573.

[19] Katz, M. L. (1987), The welfare effects of third-degree price discrimination in intermediate good markets, American Economic Review, 77, 154–167.

[20] Kim, H. and Sim, S.-G. (2015), Price discrimination and sequential contracting in monopolistic input markets, Economics Letters, 128, 39–42.

[21] Layson, S. (1994), Market opening under third-degree price discrimination, Journal of Industrial Economics, 42, 335-340.

[22] Motta, M. (2000), Competition Policy, Cambridge University Press, 2004

[23] Nahata, B., Ostaszewski, K. and Sahoo, P. K. (1990), Direction of price changes in third-degree price discrimination, American Economic Review, 80, 1254-1258.

[24] O'Brien, D. P. (2014), The welfare effects of third-degree price discrimination in intermediate good markets:the case of bargaining, RAND Journal of Economics, 45, 92–115

[25] OECD (2014), Intellectual property and standard setting, Background note by the Secretariat, DAF/COMP(2014)27.

[26] Pigou, A. C. (1920), The Economics of Welfare, Third Edition Macmillan, London.

[27] Robinson, J. (1933), Economics of Imperfect Competition, Macmillan, London.

[28] Schmalensee, R. (1981), Output and welfare implications of monopolistic third-degree price discrimination, American Economic Review, 71, 242–247.

[29] Schwartz, M. (1990), Third-degree price discrimination and output: generalising a welfare result, American Economic Review, 80, 1259–1262

[30] Sidak, G. (2013), The meaning of FRAND, Part I: Royalties, Journal of Competition Law & Economics, 9, 931–1055.

[31] Tirole, J. (1988), The Theory of Industrial Organization, Cambridge, MA: MIT Press.

[32] Varian, H. (1985), Price discrimination and social welfare, American Economic Review, 75, 870–875

[33] Yoshida, Y. (2000), Third-degree price discrimination in input market: output and welfare, American Economic Reviews, 90, 240–246