An IV framework for combining sign and long-run parametric restrictions in SVARs*

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Abstract

This paper develops a method to impose a long-run restriction in an instrumental variables (IV) framework in a SVAR which is comprised of both I(1) and I(0) variables when the shock associated with one of the I(0) variables is made transitory. This is the identification which is utilized in the small open economy SVAR that we take from the literature. The method is combined with a recently developed sign restrictions approach which can be applied in an IV setting. We then consider an alternate identification in this SVAR which makes the shocks associated with all of the I(0) variables transitory. In this case, we show that another method can be used to impose the long-run restrictions. The results from both methods are reported for the SVARs estimated with Canadian data.

Key Words: sign restrictions, long-run parametric restrictions, IV estimation, algorithms, generated coefficients, small open economy, Canada

JEL Classification: C32, C36, C51, F41

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1. Introduction

This paper shows how to impose long-run restrictions in a SVAR which consists of both I(1) and I(0) variables under different identifications for the number of transitory shocks, in an instrumental variables (IV) setting. This method is applied to a small open economy model for Canada. The model is the SVAR of Bjørnland (2009) and it consists of five variables; the foreign interest rate, output, inflation, the domestic interest rate, and the real exchange rate. Bjørnland treats the real exchange rate as an I(1) variable and the other variables as I(0). The methods in Fisher, Huh and Pagan (2016) can be used to estimate the model by IV when the long-run restrictions make all of the shocks associated with the I(0) variables transitory. However, Bjørnland makes the shock associated with only one of the I(0) variables transitory. In this case, the IV approach of Fisher, Huh and Pagan is not applicable. This paper develops an IV method to implement the long-run restriction for this case, and combines it with a recently developed approach for implementing sign restrictions.

Bjørnland (2009) identified the SVAR by imposing a full set of parametric exclusion restrictions. There is one long-run restriction and the rest are contemporaneous restrictions. The long-run restriction makes the shock associated with the interest rate transitory i.e. this shock was restricted to have a zero long-run impact on the real exchange rate.[1] The other shocks were permitted to have a long-run effect. Bjørnland's model can be estimated by utilizing a non-linear equation solver, which appears to be the method she used, or directly by maximum likelihood. In this paper, we develop a method which can impose the long-run restriction in Bjørnland's model in an IV setting.[2] It involves obtaining an estimate of the coefficient on the contemporaneous interest rate in the structural equation for the real exchange rate which enforces the long-run restriction. The advantage of this method is that it can be combined with the sign restrictions methodology of Ouliaris and Pagan (2016) so that the shocks can be identified by the long-run parametric restriction in conjunction with sign restrictions.

Because Bjørnland utilized the long-run restriction in place of a contemporaneous one, the contemporaneous response of the exchange rate to the shock associated with the interest rate is left unrestricted. If the exchange rate depreciates on impact rather than appreciates following a positive shock to the interest rate, there is an exchange rate puzzle and if it appreciates but the peak appreciation occurs after impact, there is a delayed overshooting puzzle. Both are termed 'puzzles' because the Dornbusch (1976) overshooting hypothesis predicts that the exchange rate will immediately appreciate following a positive shock to the interest rate and then it will gradually

[1] A shock is transitory if it is restricted to have a zero long-run impact on all of the I(1) variables and it is permanent if it has a long-run impact on at least one of the I(1) variables. In Bjørnland's model, there is one I(1) variable and it is the real exchange rate.

[2] The method is equivalent to maximum likelihood because the models in the paper are exactly identified.

depreciate to its pre-shock level.[3]

In this paper, the long-run restriction together with sign and contemporaneous restrictions is used to separate the shocks in the SVAR as a foreign interest rate, aggregate supply, aggregate demand, monetary policy or real exchange rate shock.[4] It is not necessary to sign restrict the response of the exchange rate to the monetary policy shock, so 'puzzles' can emerge, or to sign restrict the response of the interest rate (the policy response) to an exchange rate shock.[5] An advantage of utilizing sign restrictions is that a range of responses can be established from the sets of accepted responses. Each accepted response to a shock is equally valid under sign restrictions so it is the range of accepted responses to the shock which is of most interest, whereas full parametric identification of the shocks produces a single set of impulse responses.

We consider two identifications. The baseline identification is where the shock associated with only one of the I(0) variables is transitory. This is the case in Bjørnland's model. The alternate identification is where the shock associated with *every* I(0) variable is transitory. Under the baseline identification, the set of accepted responses shows evidence for delayed overshooting of the exchange rate by one quarter and for a systematic interest rate response by the central bank to an exchange rate shock. Under the alternate identification, the set of accepted responses shows no evidence for delayed overshooting as the peak appreciation occurs on impact. In almost all of the accepted responses, there was a systematic (positive) response of the interest rate to a (depreciating) exchange rate shock.

The treatment of the foreign and domestic interest rates and inflation as I(0) variables is appropriate on the grounds that the sample corresponds to the period of inflation targeting. It could be argued that output should be treated as an I(1) variable, rather than as an I(0) process about a linear time trend. This paper also presents the results for the structural model which treats output as an I(1) variable.

The paper has the following structure. Section 2 explains the econometric issues and methods in a simple framework that form the basis for the paper. Section 3 describes the structural model. Section 4 develops the method to implement the baseline identification and presents the algorithm from which the impulse responses are generated and judged against the sign restrictions. Results are then presented. Section 5 presents the algorithm for the alternate identification and presents the results. Section 6 discusses the results in the case where output is treated as an I(1) variable. Section 7 concludes.

2. Econometric Methods

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[3] Early studies which report these puzzles are Eichenbaum and Evans (1995) for the US and Kim and Roubini (2000) for non-US G7 countries.

[4] The sign restrictions methodology was first introduced by Faust (1998), Canova and De Nicoló (2002) and Uhlig (2005). Scholl and Uhlig (2008), using Uhlig's sign approach, find robust evidence for delayed overshooting of the US exchange rate.

[5] One interpretation of an exchange rate shock is that it is a shock to the risk premium of the currency.

The key econometric methods of the paper can be demonstrated in the context of a three variable structural system. For ease of exposition, the structural model has one lag and excludes deterministic terms. It is:

$$A_0 z_t = A_1 z_{t-1} + \varepsilon_t \tag{1}$$

where

$$z_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix},\ A_0 = \begin{bmatrix} 1 & -a_{12}^0 & -a_{13}^0 \\ -a_{21}^0 & 1 & -a_{23}^0 \\ -a_{31}^0 & -a_{32}^0 & 1 \end{bmatrix},\ A_1 = \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 \end{bmatrix},\ \varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix}$$

and the covariance matrix of the structural shocks is diagonal. Some of the variables in z_t are assumed to be I(1) and the others I(0). The shock associated with the structural equation for an I(0) variable can have a permanent effect on an I(1) variable, unless it is restricted to have a transitory effect. The first case we consider is that the structural shocks associated with the I(0) variables are transitory.

2.1. The shocks to the I(0) variables are transitory

Suppose first that y_{1t} is an I(0) variable and that y_{2t} and y_{3t} are I(1) variables.[6] Both I(1) variables enter the structural model in first difference form so that $z_t = \left[y_{1t} \quad \Delta y_{2t} \quad \Delta y_{3t} \right]'$, where $\Delta y_{it} = y_{it} - y_{it-1}$. In lag operator notation, Eq. (1) is $A(L) z_t = \varepsilon_t$ where $A(L) = A_0 - A_1 L$ and L is the operator $L z_t = z_{t-1}$. The structural moving average representation is $z_t = A(L)^{-1} \varepsilon_t = C(L) \varepsilon_t$ where $C(L) = A(L)^{-1}$ is the matrix of the responses of the variables at horizon L to the structural shocks in ε_t. It follows that $C(L)A(L) = I$, and that:

$$C(1)A(1) = I \tag{2}$$

where $C(1)$ is the matrix of the cumulative impacts of the structural shocks in the long-run and $A(1) = [A_0 - A_1]$. Writing Eq. (2) in full gives:

$$\begin{pmatrix} c_{11}(1) & c_{12}(1) & c_{13}(1) \\ c_{21}(1) & c_{22}(1) & c_{23}(1) \\ c_{31}(1) & c_{32}(1) & c_{33}(1) \end{pmatrix} \begin{bmatrix} 1 - a_{11}^1 & -a_{12}^0 - a_{12}^1 & -a_{13}^0 - a_{13}^1 \\ -a_{21}^0 - a_{21}^1 & 1 - a_{22}^1 & -a_{23}^0 - a_{23}^1 \\ -a_{31}^0 - a_{31}^1 & -a_{32}^0 - a_{32}^1 & 1 - a_{33}^1 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

[6] We assume the two I(1) variables are not cointegrated. If they are, the vector of variables in z_t can be specified as y_{1t}, the I(0) error-correction variable and y_{3t}. The case of two I(0) variables and one I(1) variable is considered below.

Note the term in square brackets on the left-hand side of Eq. (3) is $A(1)$. The solutions for $c_{21}(1)$, $c_{31}(1)$ and $c_{32}(1)$ are:

$$c_{21}(1) = \frac{(a_{31}^0 + a_{31}^1)(a_{23}^0 + a_{23}^1) + (a_{21}^0 + a_{21}^1)(1 - a_{33}^1)}{|A(1)|} \tag{4}$$

$$c_{31}(1) = \frac{(a_{32}^0 + a_{32}^1)(a_{21}^0 + a_{21}^1) + (a_{31}^0 + a_{31}^1)(1 - a_{22}^1)}{|A(1)|} \tag{5}$$

$$c_{32}(1) = \frac{(a_{31}^0 + a_{31}^1)(a_{12}^0 + a_{12}^1) + (a_{32}^0 + a_{32}^1)(1 - a_{11}^1)}{|A(1)|} \tag{6}$$

where the denominator is the determinant of $A(1)$.

The condition for ε_{1t}, the shock associated with the I(0) variable, to be transitory, i.e. to have a zero long-run impact on both of the I(1) variables, is that $c_{21}(1) = 0$ and $c_{31}(1) = 0$. It can be seen from Eqs. (4) and (5) that these two conditions will be satisfied if the two restrictions

$$a_{21}^1 = -a_{21}^0 \text{ and } a_{31}^1 = -a_{31}^0 \tag{7}$$

are placed on the structural model. The restrictions in (7) mean that the I(0) variable, y_{1t} will appear in first difference form in the structural equation for each I(1) variable.

A third restriction is required for exact identification. It could be the condition that $c_{32}(1) = 0$, so that the second shock does not have a long-run effect on the third variable. Eq. (6) shows that together with the second restriction in (7), this is achieved by setting $a_{32}^1 = -a_{32}^0$. This restriction means $\Delta^2 y_{2t}$ will appear along with Δy_{1t} on the right-hand side of the structural equation for Δy_{3t}. In other words, the first difference of all the right-hand side variables, i.e. of y_{1t} and Δy_{2t}, appear in the structural equation for Δy_{3t}, which is the formulation of Shapiro and Watson (1988).[7]

Now consider the case where we treat both y_{1t} and y_{2t} as I(0) variables so that $z_t = [y_{1t} \quad y_{2t} \quad \Delta y_{3t}]'$. The solution for $c_{31}(1)$ and $c_{32}(1)$ is the same as before. The condition

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[7] The condition $c_{23}(1) = 0$ could be used instead. This condition is achieved by imposing $a_{23}^1 = -a_{23}^0$, together with the first restriction in (7), on the structural model. This can be seen from the equation for $c_{23}(1)$, which is not shown to conserve space. In this case, the first difference of all the right-hand side variables appear in the structural equation for Δy_{2t}.

for the shocks associated with the I(0) variables to be transitory is that $c_{31}(1)=0$ and $c_{32}(1)=0$. It can be seen from Eqs. (5) and (6) that these two conditions will be satisfied if the two restrictions

$$a_{31}^{1}=-a_{31}^{0} \text{ and } a_{32}^{1}=-a_{32}^{0} \quad (8)$$

are placed on the structural model. The restrictions in (8) mean that the I(0) variables, y_{1t} and y_{2t} will appear in first difference form in the structural equation for Δy_{3t}. A further restriction, for example, $a_{12}^{0}=0$ will exactly identify the model.

We have demonstrated that if the shocks associated with the I(0) variables are all transitory, i.e. do not have a long-run effect on any of the I(1) variables, then in the structural equations for the I(1) variables, *all* of the I(0) variables appear in first difference form. A generalization of this result to cointegated systems is found in Fisher, Huh and Pagan (2016).

2.2. A shock to an I(0) variable is permanent

Staying with the case of two I(0) variables and one I(1) variable i.e. $z_t=\left[y_{1t} \quad y_{2t} \quad \Delta y_{3t}\right]'$, assume the shock associated with y_{2t} is transitory but now allow the shock associated with y_{1t} to be permanent. The condition for ε_{2t}, the structural shock in the equation for y_{2t}, to be transitory is $c_{32}(1)=0$. From Eq. (6), sufficient conditions are that

$$a_{12}^{1}=-a_{12}^{0} \text{ and } a_{32}^{1}=-a_{32}^{0} \quad (9)$$

(Note we rule out $a_{31}^{1}=-a_{31}^{0}$ because the first shock has a permanent effect on y_{3t} by assumption). The conditions in (9) say that Δy_{2t} appears as a right-hand side variable in the structural equations for both y_{1t} and Δy_{3t}. In particular, Δy_{2t} appears in the y_{1t} equation to eliminate the effect of ε_{2t} on y_{3t} in the long-run via its effect on y_{1t}. A further restriction, for example, $a_{13}^{0}=0$ will exactly identify the model.

Not only do the conditions in (9) deliver $c_{32}(1)=0$ but they also deliver $c_{12}(1)=0$ as can be seen from the solution for $c_{12}(1)$ which is $[(a_{13}^{0}+a_{13}^{1})(a_{32}^{0}+a_{32}^{1})+(a_{12}^{0}+a_{12}^{1})(1-a_{33}^{1})]/|A(1)|$. The condition that $c_{12}(1)=0$ is unappealing because it says that the *cumulated* impulse responses of y_{1t} to the second shock ε_{2t} are zero. For this to occur, there would have to be both positive and negative responses at different horizons which when summed together over a long-horizon would exactly cancel out. In practice, there would be little reason on economic grounds to impose such an identifying restriction. For this reason, we will not consider the structural model that delivers the condition $c_{32}(1)=0$ with the restrictions in (9).

It is common to combine the long-run restriction with contemporaneous zero restrictions to achieve exact identification. The parametric restrictions $a_{32}^1 = -a_{32}^0$, $a_{12}^0 = 0$ and $a_{13}^0 = 0$ exactly identify the model but they do not deliver the long-run restriction $c_{32}(1) = 0$. To see this, substitute the parametric restrictions into Eq. (6) to obtain:

$$c_{32}(1) = \frac{(a_{31}^0 + a_{31}^1)a_{12}^1}{|A(1)|} \tag{10}$$

so that $c_{32}(1) \neq 0$ unless the over-identifying restriction $a_{12}^1 = 0$ is also imposed. Another way to demonstrate this is to set $c_{32}(1) = 0$ in Eq. (6) and solve for a_{32}^0, given $a_{12}^0 = 0$, to get:

$$a_{32}^0 = -a_{32}^1 - \frac{(a_{31}^0 + a_{31}^1)a_{12}^1}{(1 - a_{11}^1)} \tag{11}$$

Eq. (11) shows that $a_{32}^1 \neq -a_{32}^0$ unless $a_{12}^1 = 0$ is also imposed in which case $c_{32}(1) = 0$, but then the model is over-identified. In a case like this, given that it is preferable to work with exactly identified models, a different method of imposing the long-run restriction is needed in the IV framework. We develop such a method in combination with sign restrictions in the next section but for now we introduce this method for the present case.

The reduced-form model underlying the structural model is $z_t = B_1 z_{t-1} + e_t$, where e_t are the VAR errors. It follows that:

$$C(1)A_0 = B(1) \tag{12}$$

where $B(1)$ is the matrix of the cumulative impacts of the VAR errors in the long-run, the elements of which are denoted $b_{ij}(1)$, and A_0 is shown in Eq. (1). Estimates of the elements of $B(1)$ are obtained from the estimated reduced-form and are denoted $\hat{b}_{ij}(1)$. Recall that the identifying restrictions we use here are $c_{32}(1) = 0$, $a_{12}^0 = 0$ and $a_{13}^0 = 0$. Substitute these into Eq. (12), and multiply the third row of $C(1)$ with the second and third columns of A_0, respectively, to obtain the equations:

$$-a_{32}^0 c_{33}(1) = b_{32}(1) \quad \text{and} \quad c_{33}(1) = b_{33}(1) \tag{13}$$

which give

$$a_{32}^0 = -\frac{b_{32}(1)}{b_{33}(1)} \tag{14}$$

This is analogous to an expression derived in Fry and Pagan (2005, Eq. (9), p.10) and in Levtchenkova, Pagan and Robertson (1998, p.512).

Fix the value of a_{32}^0 at:

$$\tilde{a}_{32}^0 = -\frac{\hat{b}_{32}(1)}{\hat{b}_{33}(1)} \tag{15}$$

The estimation of the SVAR proceeds as follows. Under the two contemporaneous zero restrictions, we regress y_{1t} on the lags of the variables and obtain the estimated residuals $\hat{\varepsilon}_{1t}$. We then estimate the equation for the third variable by regressing $\Delta y_{3t} - \tilde{a}_{32}^0 y_{2t}$ on the right-hand side variables using $\hat{\varepsilon}_{1t}$ as the instrument for y_{1t} and obtain $\hat{\varepsilon}_{3t}$. The y_{2t} equation is estimated last using $\hat{\varepsilon}_{1t}$ and $\hat{\varepsilon}_{3t}$ as instruments for y_{1t} and Δy_{3t}, respectively.

When the estimated structural coefficients are substituted into the right-hand side of Eq. (11), we obtain

$$\tilde{a}_{32}^0 = -\hat{a}_{32}^1 - \frac{(\hat{a}_{31}^0 + \hat{a}_{31}^1)\hat{a}_{12}^1}{(1-\hat{a}_{11}^1)} \tag{16}$$

where $\tilde{a}_{32}^0$ is the value given in Eq. (15). This method produces the value for a_{32}^0 which makes $c_{32}(1) = 0$ as part of the solution to the system of equations in (3) evaluated at the model's estimated coefficients and that value of a_{32}^0 is given by Eq. (15).

3. The structural model

The small open economy model of Bjørnland (2009) is estimated with Canadian data. The variables are: log output (y_t), inflation (π_t), the interest rate (i_t), the trade-weighted foreign interest rate (i_t^*), and the log of the trade-weighted real exchange rate (q_t). Output, inflation, and the foreign and domestic interest rates are treated as I(0) variables. Because output is a trending variable, a linear time trend is included as a deterministic term in the SVAR.[8] The real exchange rate is treated as an I(1) variable so it enters the SVAR in first difference form i.e. as $\Delta q_t \;\; (= q_t - q_{t-1})$.

The data for Canada is quarterly for the period 1994:Q1 to 2017:Q3. The source and construction of the data is fully described in Fisher and Huh (2016). We have updated their data, which ends in 2014:Q1, with the most recent revised trade-weights and data. Output is real GDP and the output variable enters the model as $y_t = 100[\log(GDP_t / GDP_{1994:Q1})]$ so it is a log index with a beginning

[8] The SVAR could alternatively be specified with de-trended output obtained by using a linear trend instead of output in levels. In this case, the constant is the only deterministic term in SVAR. There are no noticeable changes in the results when this specification is used instead.

value of zero. Inflation is the change in the consumer price index from the same quarter of the previous year, measured as percent per annum. The interest rate is the 3-month rate in percent per annum and the foreign interest rate is the trade-weighted average of the 3-month rates of Canada's major trading partners, in percent per annum. The real trade weighted exchange rate is defined as the number of 'Canadian' goods per unit of the 'foreign' good so that a decrease in its value represents a real appreciation of the Canadian dollar. In the calculation of the trade-weighted variables, the weight assigned to the United States averaged 75 percent over the sample so it is by far Canada's most important trading partner. Fig. 1 shows a graph of the data. The impact of the global financial crisis is clearly evident. Over 2008-2009, output, inflation, and the Canadian and foreign interest rates fell considerably, and there was a marked depreciation in the real exchange rate.

The structural model was estimated with a lag length of two as that was selected by the AIC criterion. It included a constant and a linear time trend as deterministic variables. The structural model is written as:

$$A_0 z_t = \gamma_0 + \gamma_1 t + A_1 z_{t-1} + A_2 z_{t-2} + \varepsilon_t \tag{17}$$

where

$$z_t = \begin{bmatrix} i_t^* \\ y_t \\ \pi_t \\ i_t \\ \Delta q_t \end{bmatrix}, \; A_0 = \begin{bmatrix} 1 & -a_{12}^0 & -a_{13}^0 & -a_{14}^0 & -a_{15}^0 \\ -a_{21}^0 & 1 & -a_{23}^0 & -a_{24}^0 & -a_{25}^0 \\ -a_{31}^0 & -a_{32}^0 & 1 & -a_{34}^0 & -a_{35}^0 \\ -a_{41}^0 & -a_{42}^0 & -a_{43}^0 & 1 & -a_{45}^0 \\ -a_{51}^0 & -a_{52}^0 & -a_{53}^0 & -a_{54}^0 & 1 \end{bmatrix}, \; \varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{bmatrix},$$

A_1 and A_2 are (5x5) coefficient matrices with typical element a_{ij}^1 and a_{ij}^2, respectively, and γ_0 and γ_1 are (5x1) coefficient vectors on the constant and time trend, respectively. In this model ε_{5t} is a permanent shock because it is associated with the I(1) variable. The shocks associated with the I(0) variables can either be permanent or transitory depending on the identification scheme. We now identify this model under both sign and parametric (long-run and contemporaneous) restrictions.

4. Baseline identification

In the baseline identification, there is only one shock which is transitory and that is ε_{4t}, the shock associated with the structural equation for the domestic interest rate. It is a transitory shock because it is restricted to have a zero long-run impact on the real exchange rate. This restriction, which was utilized by Bjørnland, is $c_{54}(1) = 0$. The shocks associated with the other I(0) variables can have a permanent effect i.e. a non-zero long-run impact on the real exchange rate. As there is only one long-run restriction in the baseline identification, nine restrictions are required on the coefficients in the contemporaneous matrix A_0 for exact identification. Under the baseline identification this matrix is:

$$A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -a_{21}^0 & 1 & -\overline{a}_{23}^0 & -\overline{a}_{24}^0 & -\overline{a}_{25}^0 \\ -a_{31}^0 & -a_{32}^0 & 1 & -\overline{a}_{34}^0 & -\overline{a}_{35}^0 \\ -a_{41}^0 & -a_{42}^0 & -a_{43}^0 & 1 & -a_{45}^0 \\ -a_{51}^0 & -a_{52}^0 & -a_{53}^0 & -a_{54}^0 & 1 \end{bmatrix} \tag{18}$$

Four zero contemporaneous restrictions are imposed on the structural equation for the foreign interest rate so that the foreign interest rate does not depend on contemporaneous values of the Canadian variables. The Ouliaris and Pagan (2016) approach to sign restrictions involves assigning values to some of the coefficients in A_0. Here we assign values to five of the coefficients and they are designated with a "–" above them in (18). The values are generated as follows:

$$\overline{a}_{23}^0 = \frac{\theta_1}{1-|\theta_1|},\quad \overline{a}_{24}^0 = \frac{\theta_2}{1-|\theta_2|},\quad \overline{a}_{25}^0 = \frac{\theta_3}{1-|\theta_3|},\quad \overline{a}_{34}^0 = \frac{\theta_4}{1-|\theta_4|},\quad \overline{a}_{35}^0 = \frac{\theta_5}{1-|\theta_5|} \tag{19}$$

where $\theta_i,\ i = 1,\ldots,5$ are drawn from a uniform probability density function over (–1,1) and | | denotes the absolute value. In (18), there are four zero restrictions and five generated coefficients, so that, together with the long-run restriction, the model is exactly identified.

The issue is how to implement the long-run restriction. Note that $A(1) = [A_0 - A_1 - A_2],$ as there are two lags here. Following the development in Section 2.2, we know that simply imposing the parametric restriction $a_{54}^2 = -a_{54}^0 - a_{54}^1$ on the structural equation for the real exchange rate so that the interest rate enters in first difference form will not deliver $c_{54}(1) = 0$ in the absence of further restrictions. Here we generalize the development in Section 2.2 to obtain a value for a_{54}^0 in the sign restrictions framework which enforces $c_{54}(1) = 0$ as part of the solution to $C(1)A(1) = I,$ evaluated at $\hat{A}(1) = [\hat{A}_0 - \hat{A}_1 - \hat{A}_2].$

For A_0 in (18) and for $c_{54}(1) = 0,$ the inner product of the fifth row of $C(1)$ with the fifth column of A_0 in Eq. (12) produces the equation:

$$c_{55}(1) = b_{55}(1) + \overline{a}_{25}^0 c_{52}(1) + \overline{a}_{35}^0 c_{53}(1) \tag{20}$$

Similarly, the inner product of the fifth row of $C(1)$ with the fourth column of A_0 gives:

$$a_{54}^0 = -(1/c_{55}(1))(b_{54}(1) + \overline{a}_{24}^0 c_{52}(1) + \overline{a}_{34}^0 c_{53}(1)) \tag{21}$$

Substitute Eq. (20) into Eq. (21) to obtain:

$$a_{54}^{0} = -\frac{[b_{54}(1) + \overline{a}_{24}^{0}c_{52}(1) + \overline{a}_{34}^{0}c_{53}(1)]}{[b_{55}(1) + \overline{a}_{25}^{0}c_{52}(1) + \overline{a}_{35}^{0}c_{53}(1)]} \tag{22}$$

From the reduced-form VAR, we can obtain an estimate of each element in $B(1)$ i.e. we have $\hat{b}_{ij}$. All that remains is to find an estimate of $c_{52}(1)$ and $c_{53}(1),$ since the generated coefficients are known. Denote the elements of A_0^{-1} as a_0^{ij}. It follows from Eq. (12) that:

$$\begin{pmatrix} c_{52}(1) & c_{53}(1) \end{pmatrix} = \begin{pmatrix} b_{51}(1) & b_{52}(1) & b_{53}(1) & b_{54}(1) & b_{55}(1) \end{pmatrix} \begin{bmatrix} a_0^{12} & a_0^{13} \\ a_0^{22} & a_0^{23} \\ a_0^{32} & a_0^{33} \\ a_0^{42} & a_0^{43} \\ a_0^{52} & a_0^{53} \end{bmatrix} \tag{23}$$

Once we have found estimates of the elements in the second and third columns of A_0^{-1} we can find $c_{52}(1)$ and $c_{53}(1)$. By the relationship between the reduced-form (VAR) errors and the structural shocks, given by $e_t = A_0^{-1}\varepsilon_t,$ it follows that:

$$e_{it} = a_0^{i2}\varepsilon_{2t} + \eta_{it}, \quad \eta_{it} = a_0^{i1}\varepsilon_{1t} + a_0^{i3}\varepsilon_{3t} + a_0^{i4}\varepsilon_{4t} + a_0^{i5}\varepsilon_{5t}, \quad i = 1,2,3,4,5. \tag{24}$$

and

$$e_{it} = a_0^{i3}\varepsilon_{3t} + \nu_{it}, \quad \nu_{it} = a_0^{i1}\varepsilon_{1t} + a_0^{i2}\varepsilon_{2t} + a_0^{i4}\varepsilon_{4t} + a_0^{i5}\varepsilon_{5t}, \quad i = 1,2,3,4,5. \tag{25}$$

Estimation of Eq. (24) by OLS will produce consistent estimates of a_0^{i2} because η_{it} is uncorrelated with ε_{2t} since the structural shocks are orthogonal. Similarly, estimation of Eq. (25) by OLS will produce consistent estimates of a_0^{i3} because ν_{it} is uncorrelated with ε_{3t}. These are used in Eq. (23) to give the estimates of $c_{52}(1)$ and $c_{53}(1)$ which can then used in Eq. (22) to obtain the estimate of a_{54}^{0}.

4.1. The sign restriction algorithm

First, obtain the values of the five generated coefficients in (19) by taking a draw of five θ_i coefficients. Then

(i) Estimate the foreign interest rate equation by regressing i_t^* on the lags of the variables and deterministic terms. Compute $\hat{\varepsilon}_{1t}$.

(ii) Estimate the output equation by regressing $y_t - \bar{a}_{23}^0 \pi_t - \bar{a}_{24}^0 i_t - \bar{a}_{25}^0 \Delta q_t$ on i_t^*, the lagged variables and deterministic terms, using $\hat{\varepsilon}_{1t}$ as the instrument for i_t^* and compute $\hat{\varepsilon}_{2t}$.

(iii) Estimate the inflation equation by regressing $\pi_t - \bar{a}_{34}^0 i_t - \bar{a}_{35}^0 \Delta q_t$ on i_t^*, y_t, and remaining right-hand side terms using $\hat{\varepsilon}_{1t}$ and $\hat{\varepsilon}_{2t}$ as the instruments for i_t^* and y_t, respectively. Compute $\hat{\varepsilon}_{3t}$.

(iv) Regress $\hat{e}_{it}$ on $\hat{\varepsilon}_{2t}$ and $\hat{e}_{it}$ on $\hat{\varepsilon}_{3t}$, $i = 1,..,5$ to obtain $\hat{a}_0^{i2}$ and $\hat{a}_0^{i3}$ which are used in Eq. (23) to produce $\hat{c}_{52}(1)$ and $\hat{c}_{53}(1)$. Place these in Eq. (22) to obtain the estimate of a_{54}^0, which we denote as $\tilde{a}_{54}^0$.

(v) Estimate the real exchange rate equation by regressing $\Delta q_t - \tilde{a}_{54}^0 i_t$ on i_t^*, y_t, π_t and the remaining right-hand side terms, using $\hat{\varepsilon}_{1t}$, $\hat{\varepsilon}_{2t}$ and $\hat{\varepsilon}_{3t}$ as the instruments for i_t^*, y_t and π_t respectively. Compute $\hat{\varepsilon}_{5t}$.

(vi) Estimate the interest rate equation by regressing i_t on the variables, their lags and on the deterministic terms using $\hat{\varepsilon}_{1t}$, $\hat{\varepsilon}_{2t}$, $\hat{\varepsilon}_{3t}$ and $\hat{\varepsilon}_{5t}$ as the instruments for i_t^*, y_t, π_t and Δq_t, respectively. Compute $\hat{\varepsilon}_{4t}$.

For each draw, the algorithm produces estimates of all of the coefficients in the structural model and finds $\tilde{a}_{54}^0$. Substituting $\hat{A}(1)$ into $C(1)A(1) = I$ and solving for $C(1)$ produces $\hat{c}_{54}(1) = 0$ as part of the solution. The impulse responses of the variables (in their levels) to each orthogonal shock in ε_t are calculated where the size of the shock is one standard error. The impulse responses are judged for either acceptance or rejection by the sign restrictions. The algorithm is repeated for another draw, and once a predetermined number of sets of impulse responses are accepted, no further draws are made and the algorithm terminates.

4.2. Sign restrictions on the impulse responses

In this model we separate the shocks as being a foreign interest rate (FI) shock, an aggregate supply (AS) shock, an aggregate demand (AD) shock, a monetary policy (MP) shock and a real exchange rate (RX) shock. The only shock which can be the foreign interest rate shock is ε_{1t} as it is identified by the four zero contemporaneous restrictions. Because ε_{4t} is restricted to have a zero long-run effect on the real exchange rate (a monetary neutrality assumption), it is the only candidate for the monetary policy shock. The other three shocks can be either AS, AD and RX shocks and there are 3!=6 possible attributions which can be given to the set of them.

The sign restrictions that are applied to the impulse responses are shown in Table 1 where "$\geq$" indicates a non-negative response (i.e. the variable does not fall in response to the shock), "$\leq$" indicates a non-positive response (i.e. the variable does not rise in response to the shock) and "UR" indicates an unrestricted response. They are applied to the responses which occur on impact and in the subsequent quarter. There are no sign restrictions on the responses to the foreign interest rate shock since it is identified by the contemporaneous zero restrictions. As mentioned, the transitory shock, ε_{4t}, is the only candidate for the monetary policy shock. But for it to a monetary policy shock, the responses to it must satisfy the sign restrictions for an MP shock shown in the table, which rule out "price" and "output" puzzles.

On a successful draw, the responses to ε_{2t}, ε_{3t}, and ε_{5t} satisfy the sign restrictions for one of six orders of AS, AD and RX shocks, and the responses to ε_{4t} satisfy the sign restrictions for the MP shock. If neither occurs, the draw is unsuccessful and all the impulse responses are discarded.

The sign restrictions along with the long-run zero restriction uniquely separate the AS, AD, MP and RX shocks. It is not necessary to sign restrict the contemporaneous response of the exchange rate to the monetary policy shock so an "exchange rate" puzzle can emerge, nor the contemporaneous interest rate response to the exchange rate shock. In the absence of the long-run zero restriction, it would be necessary to impose a further sign restriction to separate the MP shock from the RX shock. For example, Fisher and Huh (2016) impose the restriction that the interest rate cannot fall in response to an RX shock which depreciates the home currency, while Bjørnland and Halvorsen (2014) require the exchange rate to appreciate following a positive monetary policy shock (i.e. they rule out an "exchange rate" puzzle).

In sign restrictions, the accepted responses are arranged into ascending order *at each horizon* and summary measures are then calculated. In this paper, we will focus on the maximum and minimum response at each horizon, which provides the range of accepted responses. Following the literature, we also report the median, 16th and 84th percentile responses. Our focus though is on the range of responses because all of the accepted responses to a shock are equally valid as they are observationally equivalent. We note that each of the summary responses (including the maximum and minimum responses) at a given horizon and across horizons are almost certainly from different models i.e. they are from impulse responses that are generated from different draws of the θ_i parameters.[9]

4.3. Results

Our discussion of the results will focus on the responses to the monetary policy and real exchange rate shocks from among the set of accepted responses to all of the shocks. Recall that the exchange rate is not sign restricted to the MP shock and that the interest rate is not sign restricted to the RX shock. The algorithm continued to draw until 1,000 sets of impulse responses were accepted. We

[9] We also report the median-target responses which are calculated using the metric of Fry and Pagan (2011). This metric finds the particular draw of the θ_i parameters that minimises the distance between the accepted impulse responses and the median responses for *all* of the shocks. The median-target responses are the responses produced by this particular draw of the parameters i.e. the median-target responses come from a single model corresponding to this draw.

found the success rate (1,000 divided by the number of draws that were required to find one-thousand acceptances) to be 0.551%.

Fig. 2 shows the set of accepted responses of the variables to a monetary policy shock which raises the interest rate. For each variable, the light shaded area shows the range of responses (the region from the minimum response to the maximum response) while the dark shaded area shows the region from the 16th to the 84th percentile response. Also shown are the median and median-target responses. All of the exchange rate responses show an impact appreciation so there is no evidence of an exchange rate puzzle.[10] For most of the responses, the peak appreciation occurs one quarter after impact and that is also shown by the median and median-target responses. The peak appreciation can be as much as nearly 1.3 percent. Thereafter the responses show a gradual depreciation of the real exchange rate back to its level prior to the shock i.e. they converge to zero, consistent with the long-run restriction which made the monetary policy shock transitory. It can be seen in Fig. 2 that the accepted responses satisfy the sign restrictions for an MP shock: output and inflation fall and the interest rate rises in the current and subsequent quarter. At long horizons, the responses of output, inflation and the interest rate converge to zero as they are I(0) variables. Bjørnland (2009) also found that the peak appreciation following the MP shock occurred with a one quarter delay in Canada for her sample 1983:Q1-2004:Q4.

Fig. 3 shows the set of accepted responses of the variables to an exchange rate shock which depreciates the real value of the Canadian dollar. All of the interest rate responses show a rise in the interest rate on impact. The minimum response is for a rise of 0.05 percentage points and the maximum response for a rise of about 0.3 percentage points. The median and median target responses are somewhat below 0.2 percentage points. These results provide evidence of a systematic response of monetary policy to unexpected exchange rate movements as all of the interest rate responses have the same sign on impact (positive to a depreciating shock). In Bjørnland and Halvorsen (2014), the median response of the interest rate is positive to a depreciating exchange rate shock in Canada as here.[11] Fig. 3 also shows that output and inflation rise initially as required by the sign restrictions and then fall following the rise in the interest rate.

5. Alternate identification

The alternate identification is the one in which the shock associated with *every* equation for an I(0) variable is made transitory. Here the shock associated with the equation for the foreign interest rate, output, inflation and the interest rate is made transitory so that there are four long-run restrictions:

$$c_{51}(1)=0, \quad c_{52}(1)=0, \quad c_{53}(1)=0, \quad c_{54}(1)=0 \qquad (26)$$

The restrictions in (26) say the shocks associated with all the I(0) variables do not have a long-run

[10] Recall that a decrease in the real exchange rate (a negative response) corresponds to a real appreciation while an increase (a positive response) corresponds to a real depreciation.
[11] In their SVAR, the response of the interest rate to an exchange rate shock is not sign restricted but the response of the exchange rate to the monetary policy shock is (they rule out an exchange rate puzzle) whereas the SVAR here sign restricts neither.

effect on the real exchange rate. From the discussion in Section 2.1, these long-run restrictions are imposed when *all* the I(0) variables appear on the right-hand side of the real exchange rate equation in first difference form. Formally, it can readily be seen that if the following four parametric restrictions

$$a_{51}^{2} = -a_{51}^{0} - a_{51}^{1}, \quad a_{52}^{2} = -a_{52}^{0} - a_{52}^{1}, \quad a_{53}^{2} = -a_{53}^{0} - a_{53}^{1}, \quad a_{54}^{2} = -a_{54}^{0} - a_{54}^{1} \tag{27}$$

are imposed on $A(1) = [A_0 - A_1 - A_2]$, then the solution for $C(1)$ in $C(1)A(1) = I$ is given by (26), along with a non-zero value for $c_{55}(1)$. The restrictions in (27) mean that the real exchange rate equation (excluding the deterministic terms) becomes:

$$\begin{aligned} \Delta q_t = {} & a_{51}^{0}\Delta i_t^{*} + (a_{51}^{0} + a_{51}^{1})\Delta i_{t-1}^{*} + a_{52}^{0}\Delta y_t + (a_{52}^{0} + a_{52}^{1})\Delta y_{t-1} + a_{53}^{0}\Delta \pi_t + (a_{53}^{0} + a_{53}^{1})\Delta \pi_{t-1} \\ & + a_{54}^{0}\Delta i_t + (a_{54}^{0} + a_{54}^{1})\Delta i_{t-1} + a_{55}^{1}\Delta q_{t-1} + a_{55}^{2}\Delta q_{t-2} + \varepsilon_{5t} \end{aligned} \tag{28}$$

Now the contemporaneous coefficient matrix is:

$$A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & -a_{15}^{0} \\ -a_{21}^{0} & 1 & -\bar{a}_{23}^{0} & -\bar{a}_{24}^{0} & -a_{25}^{0} \\ -a_{31}^{0} & -a_{32}^{0} & 1 & -\bar{a}_{34}^{0} & -a_{35}^{0} \\ -a_{41}^{0} & -a_{42}^{0} & -a_{43}^{0} & 1 & -a_{45}^{0} \\ -a_{51}^{0} & -a_{52}^{0} & -a_{53}^{0} & -a_{54}^{0} & 1 \end{bmatrix} \tag{29}$$

There are three contemporaneous zero restrictions on the foreign interest rate equation as the coefficient a_{15}^{0} can be estimated. These three restrictions together with the four long-run restrictions mean that only three coefficients need to be generated for exact identification and these are shown in (29) with a "–" above them.

5.1. The sign restriction algorithm

For this identification, obtain values for the generated coefficients as before. Then

(i) Estimate the exchange rate equation, Eq. (28), using as instruments i_{t-1}^{*}, y_{t-1}, π_{t-1} and i_{t-1} for Δi_t^{*}, Δy_t, $\Delta \pi_t$ and Δi_t, respectively. Compute $\hat{\varepsilon}_{5t}$.

(ii) Estimate the foreign interest rate equation using $\hat{\varepsilon}_{5t}$ as the instrument for Δq_t and compute $\hat{\varepsilon}_{1t}$.

(iii) Estimate the output equation by regressing $y_t - \bar{a}_{23}^{0}\pi_t - \bar{a}_{24}^{0} i_t$ on the remaining right-hand side terms using $\hat{\varepsilon}_{1t}$ as the instrument for i_t^{*} and $\hat{\varepsilon}_{5t}$ as the instrument for Δq_t. Compute $\hat{\varepsilon}_{2t}$.

(iv) Estimate the inflation equation by regressing $\pi_t - \bar{a}_{34}^0 i_t$ on the remaining right-hand side terms using $\hat{\varepsilon}_{1t}$ as the instrument for i_t^*, $\hat{\varepsilon}_{2t}$ the instrument for y_t, and $\hat{\varepsilon}_{5t}$ the instrument for Δq_t. Compute $\hat{\varepsilon}_{3t}$.

(v) Estimate the interest rate equation by regressing i_t on the right-hand side terms using $\hat{\varepsilon}_{1t}$, $\hat{\varepsilon}_{2t}$, $\hat{\varepsilon}_{3t}$ and $\hat{\varepsilon}_{5t}$ as the instruments for i_t^*, y_t, π_t and Δq_t, respectively. Compute $\hat{\varepsilon}_{4t}$.

5.2. Sign restrictions on the impulse responses

Under this identification, the real exchange rate shock can only be ε_{5t} as it is the only permanent shock so there is no need for sign restrictions to be applied to it. The foreign interest rate shock is ε_{1t} as the contemporaneous zero restrictions separate it from the other transitory shocks. Sign restrictions are required to separate the transitory shocks ε_{2t}, ε_{3t} and ε_{4t} as either AS, AD or MP shocks. The sign restrictions to separate these shocks are only required on the responses of output, inflation and the interest rate and are shown in Table 1. As before, the exchange rate response to the MP shock is not restricted and the signs are applied to the impact and following quarter response.

5.3. Results

The algorithm drew until 1,000 sets of impulse responses were accepted. The success rate was 3.191%, much higher than the 0.551% found for the baseline identification. This suggests that the identifying restrictions which make the shocks associated with all of the I(0) variables transitory are more consistent with the data than the baseline identification which made only one of them transitory.

Fig. 4 shows the set of accepted responses to the monetary policy shock. All of the exchange rate responses show an impact appreciation so there is no exchange rate puzzle. The median and median-target impact appreciations are around 0.4 percent. The peak appreciation is on impact, unlike under the baseline identification, where there was a one quarter delay. After the impact appreciation, the real exchange rate depreciates and most of the responses show, over the medium term, that it has depreciated below its value before the shock (i.e. the responses are above zero in the medium term), before returning to its original value in the long-run, consistent with the long-run restriction.[12] This is not fully consistent with the Dornbusch overshooting hypothesis which predicts that after the impact appreciation the exchange rate depreciates monotonically to its initial level.

Fig. 5 shows the response of the variables to the real exchange rate shock. This shock does not depend on the values assigned to the generated coefficients, which has the implication that the

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[12] The responses of the real exchange rate to the FI, AS and AD shocks went to zero in the long-run as well, confirming empirically that the parametric restrictions on the exchange rate equation in (27) deliver the long-run restrictions in (26).

responses of each variable to it will be the same in each draw, and that is why they appear as a single response in the figure.[13] In response to an exchange rate shock which depreciates the currency, inflation and output would be expected to rise in the absence of a monetary policy response. However, the figure shows that the interest rate increases on impact which results in an immediate fall in inflation and output. Both continue to fall for several quarters before gradually returning to their initial levels in line with the falling interest rate.

6. The structural model with output I(1)

Given that our sample corresponds to the period of inflation targeting, the treatment of inflation and interest rates as I(0) variables would appear justified. A justification for treating output as an I(0) variable about a linear time trend is that our sample also covers the period of the 'great moderation', though it does cover the global financial crisis period. Nevertheless, it is plausible to treat output as an I(1) variable. In this section, we investigate the results when output enters the model in first difference form, under the baseline and alternate identifications i.e. when

$$z_t = \begin{bmatrix} i_t^* & \Delta y_t & \pi_t & i_t & \Delta q_t \end{bmatrix}'.$$

6.1. Baseline identification

Under the baseline identification, the monetary policy shock is a transitory shock which means it has a zero long-run impact on the real exchange rate *and* output as both are I(1) variables. The requirement for the monetary policy shock to be transitory is $c_{24}(1)=0$ and $c_{54}(1)=0$. However, our method cannot accommodate this case because the two parameters of interest in A_0, namely, a_{24}^0 and a_{54}^0, cannot be solved for as the implied equations in (12) are not recursive. This demonstrates that our approach is not general. Nevertheless, it can be extended to identification structures which yield a system of equations in (12) which can be solved recursively.

We now consider the case of one long-run restriction, namely, that the monetary policy shock has a zero long-run impact on the real exchange. We treat this as the baseline case here even though the monetary policy shock is a permanent shock as it can have a non-zero long-run effect on output. For this case, the algorithm does not change except that Δy_t replaces y_t in the equations. The success rate for 1,000 acceptances is somewhat larger at 0.854%, compared to 0.551%, reported in section 5.3, but it is still small. Fig. 6 shows the set of accepted responses to the MP shock where it can be seen that many of the exchange rate responses (34% of them) now show an impact depreciation i.e. have an exchange rate puzzle. Furthermore, almost all of the output responses show that the monetary policy shock reduces output permanently, which is not consistent with the prediction in standard macroeconomic models of monetary neutrality. Fig. 7 shows the set of accepted responses to an RX shock that depreciates the currency. Almost all (92%) of the interest

[13] Because ε_{5t} does not change from draw to draw, the fifth column of A_0^{-1} does not change either as an equation analogous to Eq. (24) or Eq. (25) in the text will confirm. Therefore, the responses of the variables to the fifth (real exchange rate) shock do not change from one draw to the next.

rate responses show a rise on impact so the earlier finding of a systematic response of monetary policy to an exchange rate shock is robust to this variation in the structural model.

6.2. Alternate identification

Under the alternate identification, the shock associated with the equation for every I(0) variable is made transitory. In this case, the shock associated with the inflation equation, and the foreign and domestic interest rate equations, is restricted to have a zero long-run effect on output and the real exchange rate so there are six long-run zero restrictions. The three with respect to output are

$$c_{21}(1) = 0, \quad c_{23}(1) = 0, \quad c_{24}(1) = 0 \tag{30}$$

and the three with respect to the real exchange rate are

$$c_{51}(1) = 0, \quad c_{53}(1) = 0, \quad c_{54}(1) = 0 \tag{31}$$

We know that imposing the following three parametric restrictions on the output equation

$$a_{21}^2 = -a_{21}^0 - a_{21}^1, \quad a_{23}^2 = -a_{23}^0 - a_{23}^1, \quad a_{24}^2 = -a_{24}^0 - a_{24}^1 \tag{32}$$

and the following three on the exchange rate equation

$$a_{51}^2 = -a_{51}^0 - a_{51}^1, \quad a_{53}^2 = -a_{53}^0 - a_{53}^1, \quad a_{54}^2 = -a_{54}^0 - a_{54}^1 \tag{33}$$

will deliver the six long-run restrictions. The output equation then has the first difference of all the I(0) variables together with Δq_t and Δq_{t-1} as right hand side variables. Similarly, the real exchange rate equation has the first difference of all the I(0) variables, along with Δy_t and Δy_{t-1} as right-hand side variables. To separate the two equations from each another, the coefficient on Δq_t in the output equation is generated i.e. it is $\overline{a}_{25}^0$, and to separate the inflation equation from the interest rate equation the coefficient a_{34}^0 is generated i.e. it is $\overline{a}_{34}^0$. To complete the exact identification, we set $a_{13}^0 = 0$ and $a_{14}^0 = 0$, which separates the foreign interest rate equation from the other equations and identifies ε_{1t} as the foreign interest rate shock.

The sign restrictions are used to separate ε_{2t} and ε_{5t} as either AS or RX shocks, and ε_{3t} and ε_{4t} as either AD or MP shocks. If at least one pair turns out to be neither, the draw is unsuccessful. On each draw of the two generated coefficients, the model is estimated by IV, the impulse responses found and judged by the sign restriction. For this model, not one set of impulse responses satisfied the sign restrictions for the shocks in 1.5 million draws. We attribute this finding to the highly constrained nature of the identification which imposes six long-run restrictions, two more than under the alternate identification where output was treated as an I(0) variable.

7. Conclusion

This paper develops a method to combine a long-run restriction with sign restrictions in a model where the shock associated with one of the I(0) variables is transitory, which is the case in the SVAR of Bjørnland (2009). The method can be generalized to other long-run identifications provided the identifying assumptions are such that the implied system of equations permit the parameters of interest to be solved for recursively. As such, the method is not completely general and its suitability will depend on the application. We show that the method in Fisher, Huh and Pagan (2016) can be utilized to make the shock associated with every I(0) variable transitory.

In the SVAR, the baseline identification is where the shock associated with one of the I(0) variables is transitory (the monetary policy shock) and the alternate identification is where the shocks associated with all of the I(0) variables are transitory. There is evidence for delayed overshooting of the Canadian exchange rate by one quarter under the baseline identification but not under the alternate identification where the peak exchange rate response occurs on impact. An exchange rate puzzle only emerges in the model where output is treated as I(1) but here almost all of the accepted responses to the monetary policy shock have a long-run effect on output, which is hard to reconcile with standard models in macroeconomics. An empirical finding that emerges, consistently across structural models and identifications, is that there is a systematic response of the (policy) interest rate to exchange rate shocks in Canada.

References

Bjørnland, H.C., 2009. Monetary policy and exchange rate overshooting: Dornbusch was right after all. Journal of International Economics 79(1), 64-77.

Bjørnland, H.C., Halvorsen, J.I., 2014. How does monetary policy respond to exchange rate movements? New international evidence. Oxford Bulletin of Economics and Statistics 76(2), 208-232.

Canova, F., De Nicoló, G., 2002. Monetary disturbances matter for business fluctuations in the G-7. Journal of Monetary Economics 49(6), 1131-1159.

Dornbusch, R., 1976. Expectations and exchange rate dynamics. Journal of Political Economy 84(6), 1161-1176.

Eichenbaum, M., Evans, C.L., 1995. Some empirical evidence on the effects of shocks to monetary policy on exchange rates. Quarterly Journal of Economics 110(4), 975-1009.

Faust J., 1998. The robustness of identified VAR conclusions about money. Carnegie-Rochester Conference on Public Policy 49, 207-244.

Fisher L.A., Huh, H-S., 2016. Monetary policy and exchange rates: Further evidence using a new method for implementing sign restrictions. Journal of Macroeconomics 49, 177-191.

Fisher L.A., Huh, H-S., Pagan, A.R., 2016. Econometric methods for modelling systems with a mixture of I(1) and I(0) variables. Journal of Applied Econometrics 31(5), 892-911.

Fry R., Pagan, A.R., 2005. Some issues in using VARs for macroeconometric research. Centre for Applied Macroeconomic Analysis, Australian National University, CAMA Working Paper 19/2005.

Fry R., Pagan, A.R., 2011. Sign restrictions in structural vector autoregressions: A critical review. Journal of Economic Literature 49(4), 938-960.

Kim, S., Roubini, N., 2000. Exchange rate anomalies in the industrial countries: A solution with a structural VAR approach. Journal of Monetary Economics 45(3), 561-586.

Levtchenkova, S., Pagan, A.R., Robertson, J.C., 1998. Shocking Stories. Journal of Economic Surveys 12(5), 507-532.

Ouliaris, S., Pagan, A.R., 2016. A method for working with sign restrictions in structural equation modelling. Oxford Bulletin of Economics and Statistics 78(5), 605-622.

Scholl, A., Uhlig, H., 2008. New evidence on the puzzles: Results from agnostic identification on monetary policy and exchange rates. Journal of International Economics 76(1), 1-13.

Shapiro, M.D., Watson, M.W., Sources of business cycle fluctuations. In: Fischer, S. (Ed.), NBER Macroeconomics Annual 1988, vol. 3, pp. 111-148.

Uhlig, H., 2005. What are the effects of monetary policy on output? Results from an agnostic identification procedure. Journal of Monetary Economics 52(2), 381-419.

Table 1. Sign restrictions

Shock\Variable	GDP	Inflation	Interest Rate	Real Exchange Rate
AS	≥ 0	≤ 0	UR	UR
AD	≥ 0	≥ 0	≥ 0	≤ 0
MP	≤ 0	≤ 0	≥ 0	UR
RX	≥ 0	≥ 0	UR	≥ 0

Notes: AS denotes an aggregate supply shock, AD an aggregate demand shock, MP a monetary policy shock and RX a real exchange rate shock. The designation "$\geq$" indicates a non-negative response so that the variable does not fall in response to the shock while "$\leq$" indicates a non-positive response so that the variable does not rise in response to the shock. UR denotes an unrestricted response of the variable to the shock. The sign restrictions are imposed on the impact response and on the response for the following quarter.

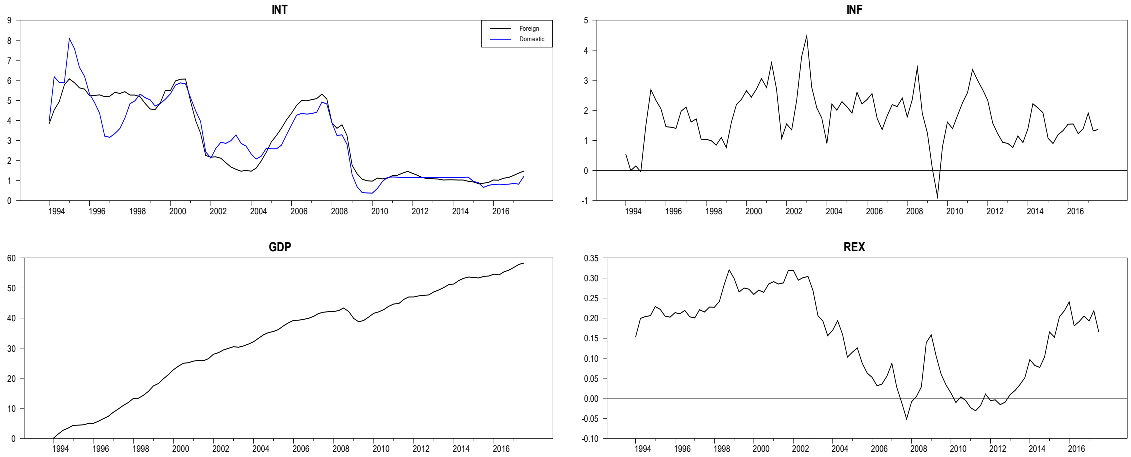

Fig. 1. Graph of the Canadian data

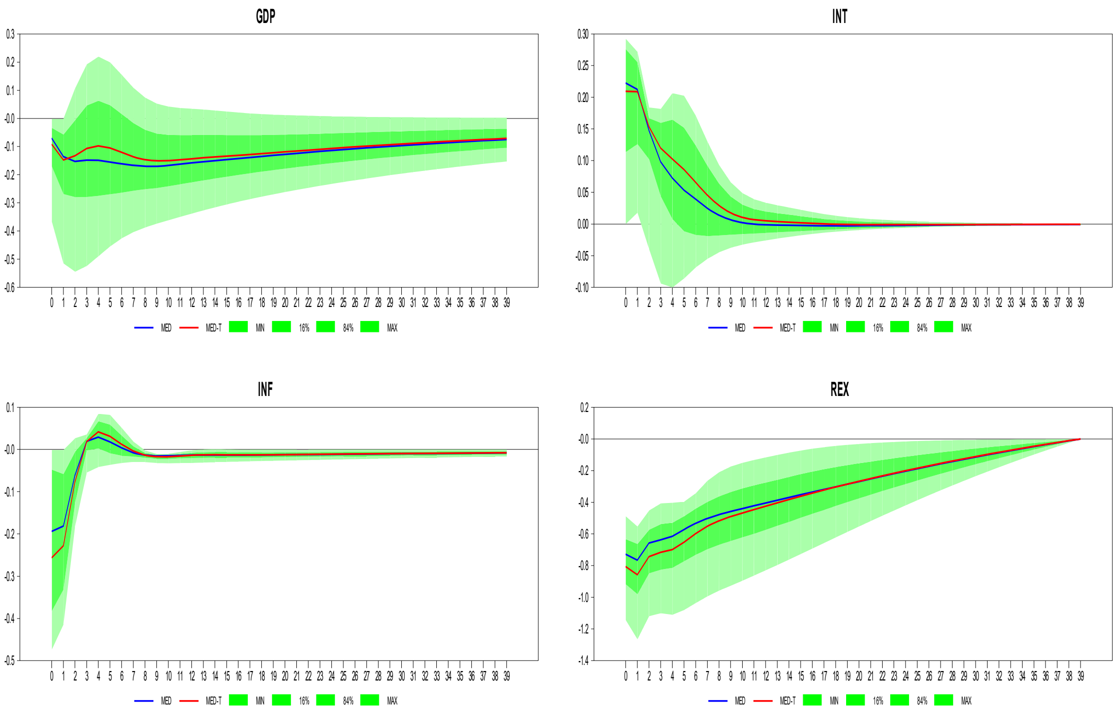

Fig. 2. Response of variables to monetary policy shock: Baseline identification

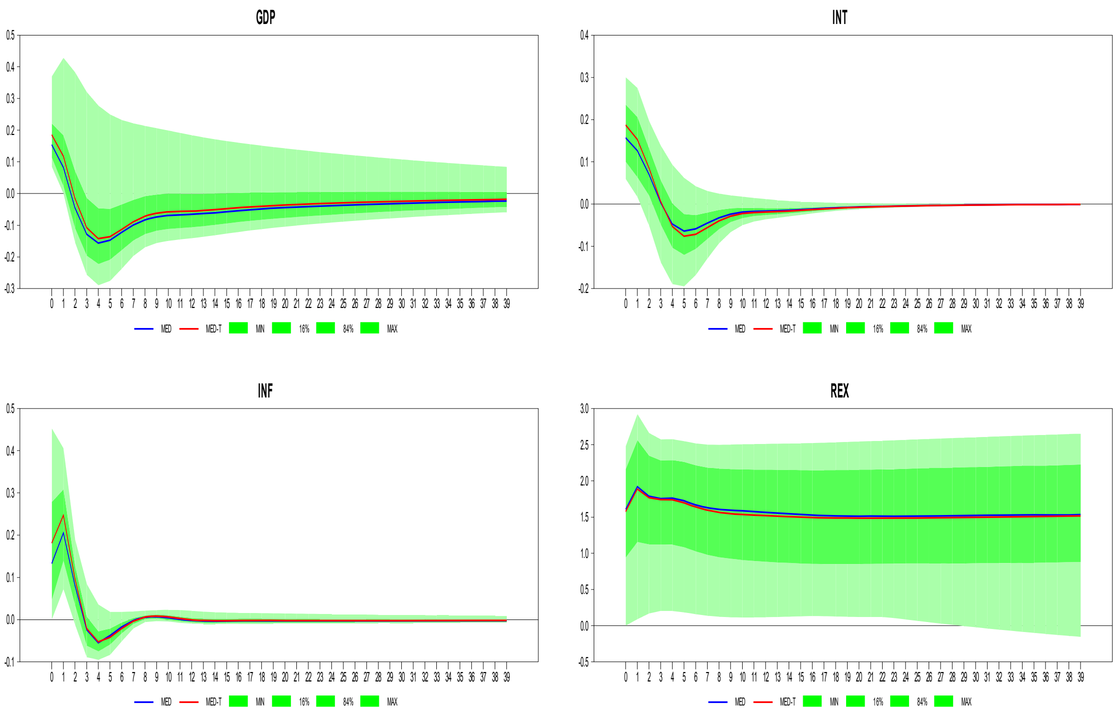

Fig. 3. Response of variables to real exchange rate shock: Baseline identification

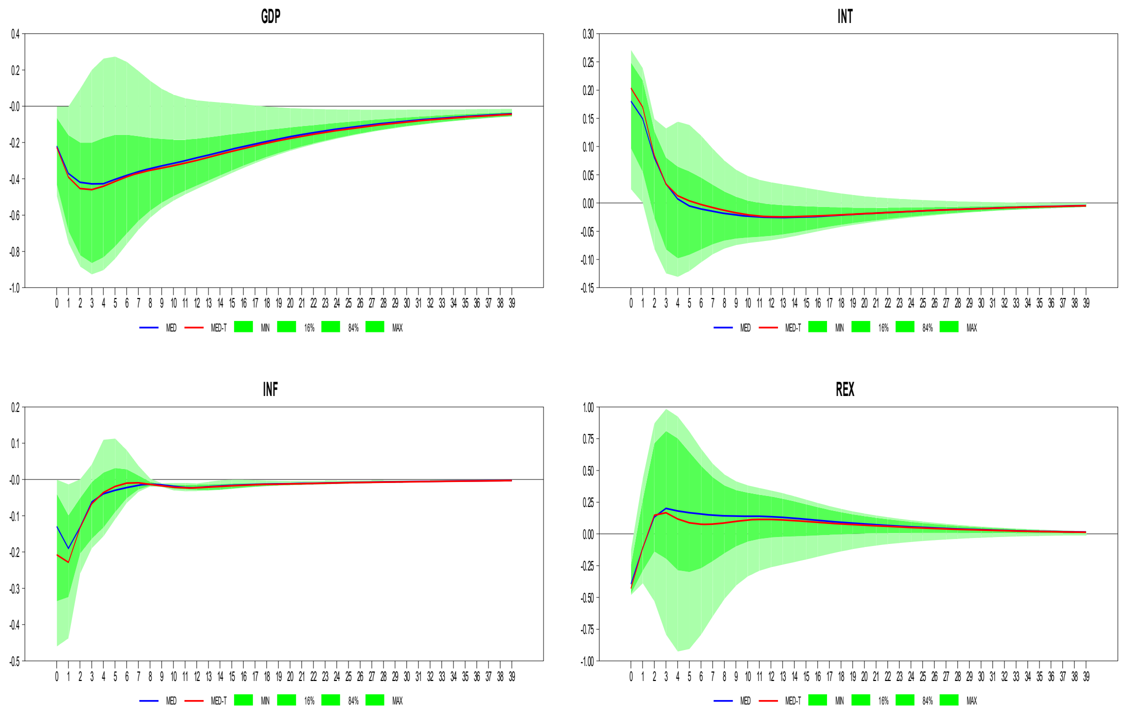

Fig. 4. Response of variables to monetary policy shock: Alternate identification

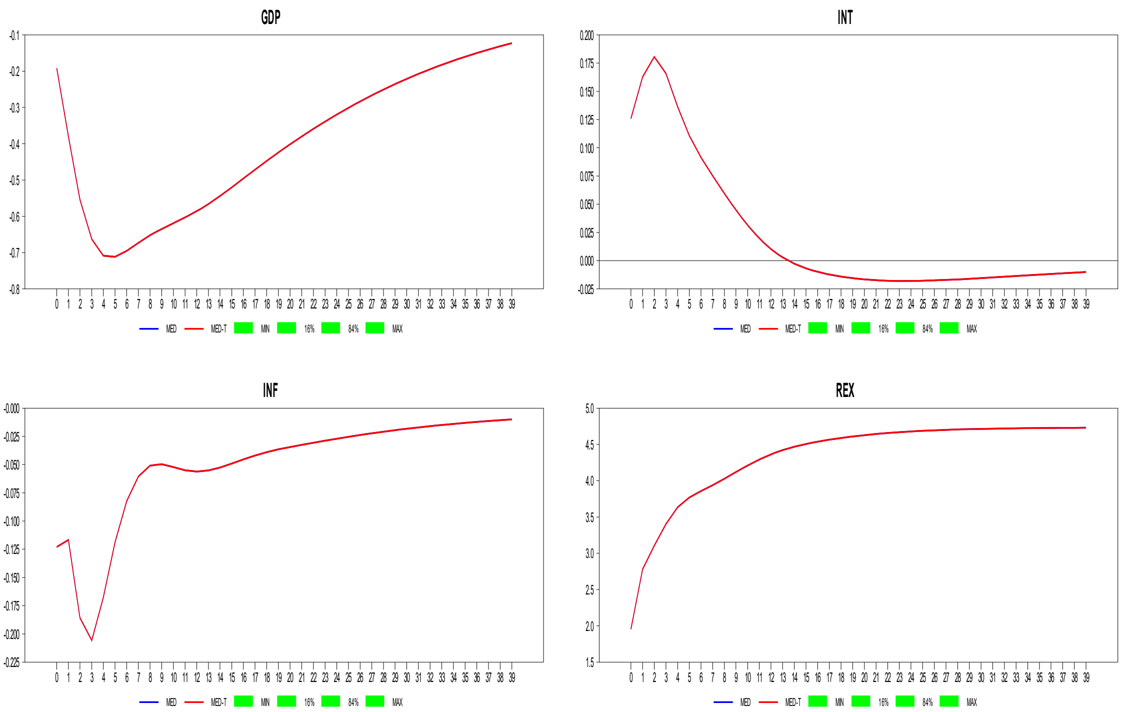

Fig. 5. Response of variables to real exchange rate shock: Alternate identification

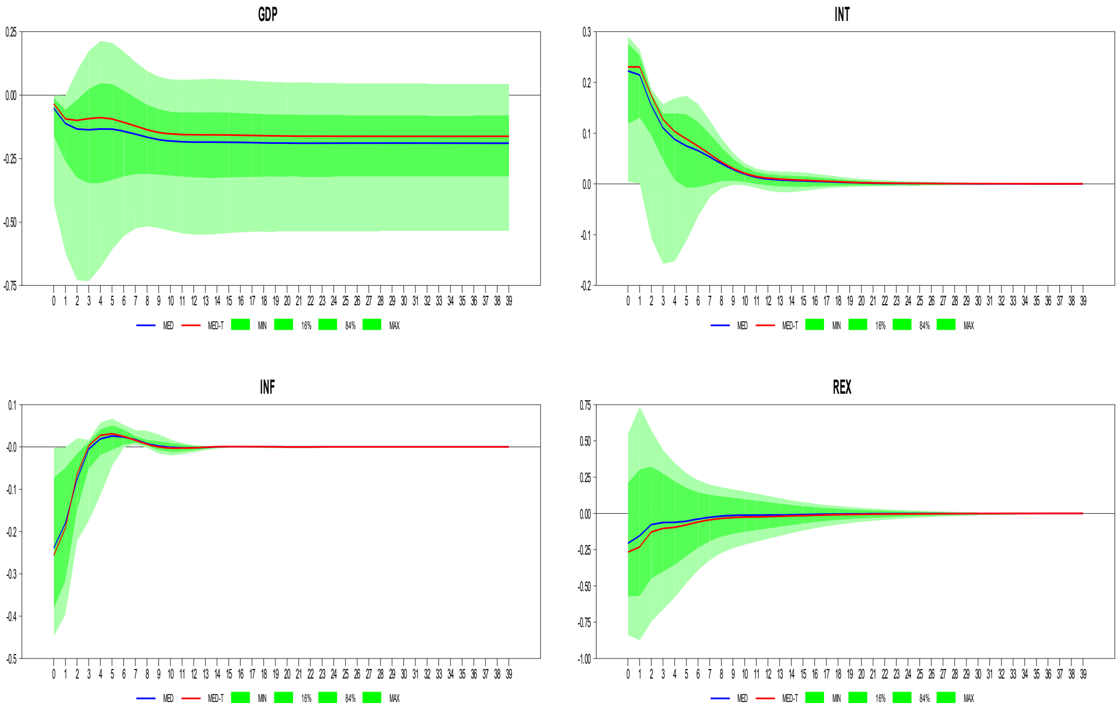

Fig. 6. Response of variables to monetary policy shock: Baseline identification with output I(1)

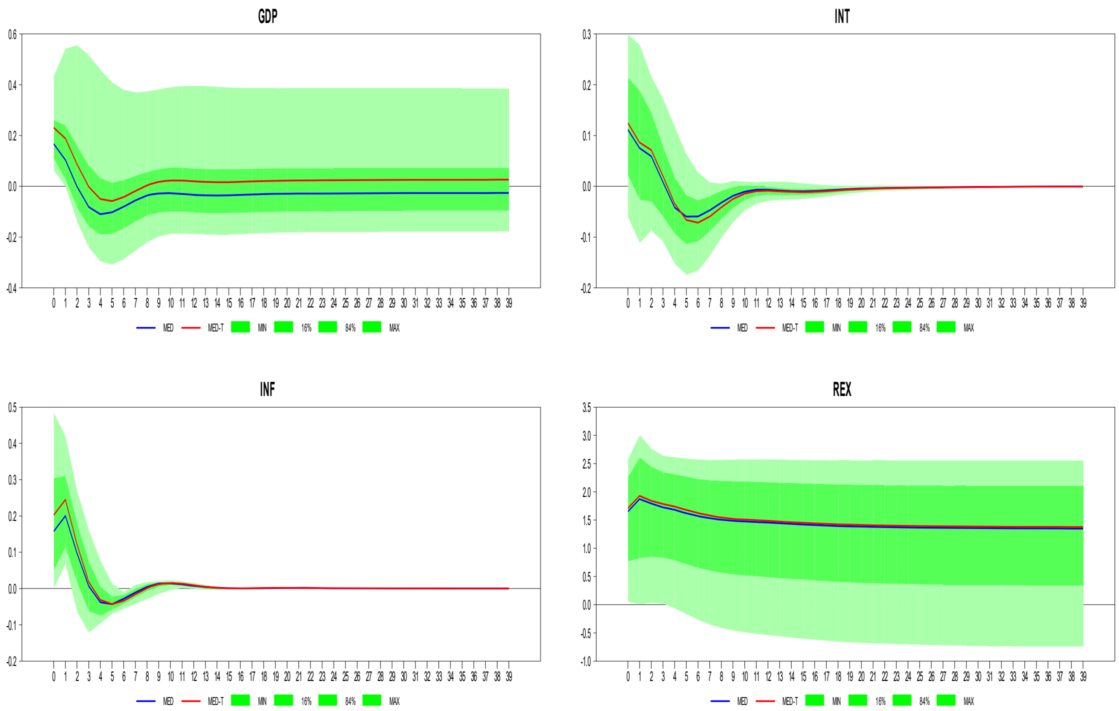

Fig. 7. Response of variables to real exchange rate shock: Baseline identification with output I(1)