Testing for Structural Breaks in Return-Based Style Regression Models[1]

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Abstract: It is important for investors to know not only the style of a fund manager in whom they are interested, but also whether this style is constant or changing through time. The style is now easily identified by the so-called style regression. However, there has been no formal and statistically valid method to test for a change in manager style when the two typically imposed restrictions (sum-to-one and non-negativity) are jointly present in style analysis. In this study, we apply and extend the results of Andrews (1997a, 1997b, 1999, 2000) to develop a valid testing procedure for the possibility wherein the location of any possible change does not need to be specified and the case of multiple shifts is accommodated. When our proposed test is applied to the Fidelity Magellan Fund, it is revealed that the fund's style changed at least twice between 1988 and 2017.

Key Words: Structural Shift; Boundary Parameter; Maximum Chow Test; Style Regression

JEL classifications: C12, C18

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1. Introduction

Analysis of asset manager style based on Sharpe's (1988, 1992) concept of effective mix has become one of the most widespread and influential analytic practices in the investment banking and pension fund management industries. The style analysis, often called "return-based style analysis" (RBSA hereafter), is obtained from constrained regression of the returns of a mutual fund on appropriately chosen style indices. The imposed constraints consist of two restrictions: (i) sum of all factor exposures is equal to one (sum-to-one restriction), and (ii) exposures should be non-negative (non-negativity restriction). Its appeal derives from its conceptual elegance, its ease of interpretation, and ready availability of input data such as style indices for RBSA.

Although RBSA has been popular in analyzing manager style, its one important limitation is the assumption that manager style does not change over time, which unrealistically implies that the manager always adopts the same investment strategy. However, many studies have presented empirical evidence supporting that manager changes over time (e.g., Gallo and Lockwood, 1999; Kim et. al., 2000; Posthuma and Van Der Sluis, 2005).

When shift in manager style is assumed to occur continuously and gradually over time, such smoothing time variation can be captured by "time-varying parameter models" such as state-space models usually estimated by the Kalman filter method. Several studies including Swinkels and Van Der Sluis (2006), Darolles and Vaissie (2012), and Marquesa et. al. (2012) proposed such modeling methods to incorporate smoothing time variation in style analysis. Although these studies opened a path toward a more general framework for style analysis, their methodology is not fully general as their time-varying models can only be applicable to either the weak style analysis or semi-strong style analysis according to the terminology of DeRoon et. al. (2004). In their study, RBSA subject to both sum-to-one and non-negativity restrictions is called "strong RBSA," while RBSA without any restriction is called "weak RBSA." Additionally "semi-strong RBSA" is subject to only sum-to-one restriction (no non-negativity restriction).

There can be a variety of sources generating shifts and time variation in manager style. Two obvious candidates are (i) mangers' investment philosophy and (ii) management structure. If these factors are considered important sources, then the previously proposed "time-varying parameter models" become inappropriate, because these factors do not change continuously and gradually. Rather they tend to change infrequently and abruptly. In that case, the standard structural break models pioneered by Chow (1960), Andrews (1993), Bai (1997), and Bai and Perron (1998) can provide more realistic ways to capture time variation in style analysis. To the best of our knowledge, only two studies applied Chow-type structural break tests to style analysis. Swinkels and Van Der Sluis (2001) used the standard Chow test to detect structural breaks in style analysis, but with *a priori* imposed break dates. The limitation of imposing known break dates was relaxed by Annaert and Van Campenhout (2002) following the methodology of Andrews (1993), Bai (1997), and Bai and Perron (1998). However, their testing procedure with unknown break dates can be applied only to either weak RBSA or semi-strong RBSA.

It is remarkable, therefore, that no generally applicable and rigorously justified methods presently exist to efficiently test for changes in manager style when the two restrictions (sum-to-one and non-negativity) are jointly imposed. The explanation for this is, however, straightforward: the essential non-negativity constraints on the coefficients of the style regression equation create a particularly difficult non-standard problem for inference, as explained in Andrews (1997a, 1997b, 1999, 2000). Previously, a generally applicable theory of inference in such situations, especially when the true parameter may be on the boundary imposed by the non-negativity constraints, was not available. Andrews (1999) provided general and elegant results that, for the first time, make possible the desired inferences in such circumstances. Kim et. al. (2005) applied Andrews's results to construct confidence intervals of style weights. In this study, we apply and extend Andrews's results to develop a formal structural break test in the framework of Sharpe's effective mix when the two restrictions (sum-to-one and non-negativity) are imposed jointly.

We are particularly motivated to obtain methods that permit us to investigate whether the style of a fund may have changed through time, as shifts in style represent a form of event risk that investors may prefer to avoid. A fund manager's style can appear to evolve or shift, or a fund may experience manager turnover, leading to apparent changes in style. Are such apparent changes real, or do they simply represent random fluctuations around a relatively stable core style? We will provide straightforward procedures, based on Andrews′s (1997a, 1997b, 1999, 2000) results, that will permit us to answer these and related questions.

The paper is organized as follows. In Section 2, we provide a brief review of the relevant aspects of style analysis. We then develop a new testing procedure for testing for one or more style shifts at known points in time in the presence of maintained sum-to-one and non-negativity constraints. In Section 3, we extend Andrews′s (1997a, 1997b, 1999, 2000) results to develop a test for a style shift at an *unknown* point in time. Section 4 discusses extensions to test for multiple style shifts at unknown points in time. In Section 5, we apply the proposed method to real data. Section 6 concludes.

2. Sharpe's Effective Mix

2.1 A Brief Review

Sharpe's effective mix provides a way to analyze the investment style of a fund by relating fund performance to a specified set of relevant investment style indices. The relationship between fund performance and style index performance is as follows:

$$y_t = X_t'\beta_0 + \varepsilon_t, \quad t = 1, 2, \ldots, T,$$

where y_t denotes fund returns in period t, X_t is a $k \times 1$ vector of style index returns in period t, β_0 is an unknown $k \times 1$ vector of style weights for the fund, and ε_t is the fund's idiosyncratic return, orthogonal to the style indices in the sense that $E(X_t \varepsilon_t) = 0$. The style weights must satisfy two key conditions: they must (i) sum to one $(\iota'\beta_0 = 1)$ and (ii) be non-negative $(\beta_0 \geq 0)$. These restrictions ensure that we interpret $X_t'\beta_0$ as the return on a "style portfolio" wherein short positions in the style indices are not permitted.

A primary goal of Sharpe's effective mix analysis is to determine the unknown style weights, β_0. This can be accomplished using standard constrained least squares regression methods. Estimated style weights satisfying the necessary constraints can be obtained by solving the constrained least squares problem:

$$\min_{\beta} \; T^{-1} \sum_{t=1}^{T} (y_t - X_t'\beta)^2 ,$$
$$s.t. \quad \iota'\beta = 1, \quad \beta \geq 0 .$$

This is a convex linear-quadratic programming problem. Under general conditions, the resulting estimator $\tilde{\beta}$ is a strongly consistent estimator for β_0, as proved by Andrews (1997a, 1997b, 1999, 2000).

2.2 Style Shifts and Hypotheses about Style Weights

The starting point for testing whether manager style has shifted is to cast the hypotheses about manager style into a framework that permits us to apply Andrews's (1997a, 1997b, 1999, 2000) results. To construct a test with power against style shifts, we elaborate the basic style regression and consider

$$\begin{aligned} y_t &= \tilde{X}_{1t}'\beta_1^* + \tilde{X}_{2t}'\beta_2^* + \varepsilon_t \\ &\equiv \tilde{X}_t'\beta^* + \varepsilon_t, \end{aligned}$$

where $\tilde{X}_{1t} = X_t$ in periods up to time $[\tau T]$ (assumed known for now) and the zero vector thereafter and $\tilde{X}_{2t} = X_t$ for $t > [\tau T]$ and zero otherwise. Moreover, $\beta^{*\prime} = \left(\beta_1^{*\prime} : \beta_2^{*\prime}\right)$. Here, τ is between zero and one, and $[\tau T]$ is the integer part of τT. Hereafter, we will use τT in place of $[\tau T]$ when there is no confusion. The relevant null thus has the form $\beta_1^* = \beta_2^*$, which we re-write in the standard form as

$$S\beta^* = 0,$$

where S is a $k \times 2k$ matrix given by $S \equiv [I_k | -I_k]$. Here S corresponds to R of the general textbook setup of $R\beta^* = r$, and we have $r = 0$. The alternative is that $\beta_1^* \neq \beta_2^*$ (or $S\beta^* \neq 0$); that is, that style differs before and after period τ.

For the general situation, the test is based on $R\tilde{\beta}_T - r$. To analyze style shifts, we thus consider tests based on $S\tilde{\beta}_T = \tilde{\beta}_{1T} - \tilde{\beta}_{2T}$, where $\tilde{\beta}_T = (\tilde{\beta}_{1T}', \tilde{\beta}_{2T}')$ is the solution to the constrained regression:

$$\begin{aligned} &\min_{\beta_1,\, \beta_2} \quad T^{-1}\sum_{t=1}^{T}(y_t - X_{1t}'\beta_1 - X_{2t}'\beta_2)^2, \qquad (2.1) \\ &s.t. \quad \iota'\beta_1 = 1,\ \beta_1 \geq 0, \\ &\qquad\quad \iota'\beta_2 = 1,\ \beta_2 \geq 0. \end{aligned}$$

In the usual situation wherein the non-negativity constraints are absent, the standard statistics based on $R\hat{\beta}_T - r$ (where $\hat{\beta}_T$ is the ordinary least squares [OLS] estimator) have a chi-squared distribution asymptotically (see, e.g., Chapter 4; in White, 1984). The standard test based on $S\hat{\beta}_T$ is well known, such as the Chow test for "structural breaks" (Chow, 1960).

The presence of the non-negativity constraints renders the standard theory inapplicable. However, Andrews (1997a, 1997b, 1999, 2000) proposed methods that permit computation of test statistics whose level can be controlled asymptotically, making it possible, for the first time, to conduct valid tests of hypotheses for the style weights of Sharpe's effective mix and, particularly, for testing for shifts in manager style.

Applying Andrews′s (1997a, 1997b, 1999, 2000) methods, one can show under the null that for the general case,

$$T(R\tilde{\beta}_T - r)'\Gamma^{-1}(R\tilde{\beta}_T - r) - \tilde{\lambda}_T' R' \Gamma^{-1} R\tilde{\lambda}_T \xrightarrow{p} 0,$$

where $\xrightarrow{p}$ denotes convergence in probability, Γ is a given $k \times k$ non-singular matrix,[1] and $\tilde{\lambda}_T$ is a $k \times 1$ random vector whose asymptotic distribution can be straightforwardly characterized and well approximated. This implies that the asymptotic distribution of $\tilde{\lambda}_T' R' \Gamma^{-1} R\tilde{\lambda}_T$ is also well approximated. The asymptotic equivalence lemma (e.g., Lemma 4.7 in White, 1984) implies that the computable statistic $T(R\tilde{\beta}_T - r)'\Gamma^{-1}(R\tilde{\beta}_T - r)$ has the same distribution as $\tilde{\lambda}_T' R' \Gamma^{-1} R\tilde{\lambda}_T$ asymptotically, from which we can construct *p*-values for testing the null, $R\beta^* = r$.

Andrews′s (1997a, 1997b, 1999, 2000) results imply that for the generic case with $\iota'\beta = 1,\ \beta \geq 0,$ the crucial random vector $\tilde{\lambda}_T$ is the solution to the following problem:

$$\min_{\lambda}\ (\lambda - \hat{Z}_T)'\hat{M}_T(\lambda - \hat{Z}_T), \tag{2.2}$$
$$s.t. \quad \iota'\lambda = 0, \quad Q\lambda \leq 0$$

[1] The weighting matrix Γ is not necessarily deterministic. We can allow Γ to be a random matrix (denoted by $\tilde{\Gamma}_T$), which depends on both data and sample size as long as it converges in probability to a non-stochastic, positive-definite matrix. A common example for $\tilde{\Gamma}_T$ is given by $\tilde{\Gamma}_T = R\tilde{D}_T R',$ where $\tilde{D}_T = \tilde{M}_T^{-1}\tilde{V}_T\tilde{M}_T^{-1}$ and $\tilde{M}_T$, $\tilde{V}_T$ are defined later. One can show that $\tilde{\Gamma}_T$ converges to a non-stochastic, positive-definite matrix in probability. This particular weighting matrix is known to be the optimal weighting matrix in the case without non-negativity and sum-to-one restrictions. In the present analysis, this choice is not known to be optimal. Finding an optimal weighting matrix in this case to maximize power would be an interesting issue, which we leave for future research.

where $\hat{M}_T \equiv T^{-1}\sum_{t=1}^{T} X_t X_t'$, $\hat{Z}_T \equiv \hat{M}_T^{-1}\hat{G}_T$, $\hat{G}_T$ is a $k \times 1$ multivariate normal random vector, $\hat{G}_T \sim N(0, \hat{V}_T)$, $\hat{V}_T \equiv T^{-1}\sum_{t=1}^{T} X_t\tilde{\varepsilon}_t\tilde{\varepsilon}_t' X_t'$, $\tilde{\varepsilon}_t \equiv y_t - X_t'\tilde{\beta}_T$, and $Q = [q_{ij}]$ is a matrix identifying the elements of β^* that are "known" to satisfy the boundary condition, $\beta_j^* = 0$.

The matrix Q has l rows (one for each element of β^* "known" to be zero) and k columns (one for each element of β^*). The elements q_{ij} of Q are zero, except when the ith "zeroed" element of β^* has index j, in which case q_{ij} = -1. Fortunately, we are not required to have exact *a priori* knowledge about which elements of β^* are zero. We can acquire this knowledge by running a preliminary unconstrained least squares regression and identifying elements β_j^* satisfying the boundary condition as those whose associated $t-$statistics are insignificant (at a level tending to zero as $T \to \infty$).

Because we can compute a large number (denoted by N) of Monte Carlo realizations of $\tilde{\lambda}_T$ by solving (2.2) for a large number of realizations of $\hat{G}_T$, we can build up a Monte Carlo estimate of the distribution of $\tilde{\lambda}_T' R' \Gamma^{-1} R \tilde{\lambda}_T$ and therefore of our test statistic $T(R\tilde{\beta}_T - r)'\Gamma^{-1}(R\tilde{\beta}_T - r)$. Comparing $T(R\tilde{\beta}_T - r)'\Gamma^{-1}(R\tilde{\beta}_T - r)$ with the quantiles of $\tilde{\lambda}_T' R' \Gamma^{-1} R \tilde{\lambda}_T$ yields the desired *p*-value.

To test specifically for style shifts, we replace $R\tilde{\beta}_T - r$ with $S\tilde{\beta}_T = \tilde{\beta}_{1T} - \tilde{\beta}_{2T}$ and $R\tilde{\lambda}_T$ with $\tilde{\lambda}_{1T} - \tilde{\lambda}_{2T}$, where now $\tilde{\lambda}_T = (\tilde{\lambda}_{1T}', \tilde{\lambda}_{2T}')'$ solves the problem:

$$\min_{\lambda} \ (\lambda - \tilde{Z}_T)'\tilde{M}_T(\lambda - \tilde{Z}_T), \tag{2.3}$$

$$s.t. \quad \iota'\lambda_1 = 0, \ \iota'\lambda_2 = 0, \ \tilde{Q}\lambda \le 0,$$

where $\tilde{M}_T \equiv T^{-1}\sum_{t=1}^{T} \tilde{X}_t \tilde{X}_t'$, $\tilde{Z}_T \equiv \tilde{M}_T^{-1}\tilde{G}_T$, $\tilde{G}_T$ is a $2k \times 1$ multivariate normal random vector $\tilde{G}_T \sim N(0, \tilde{V}_T)$, $\tilde{V}_T \equiv T^{-1}\sum_{t=1}^{T} \tilde{X}_t\tilde{\varepsilon}_t\tilde{\varepsilon}_t'\tilde{X}_t'$, $\tilde{\varepsilon}_t \equiv y_t - \tilde{X}_t'\tilde{\beta}_T$, and $\tilde{Q}$ = [$\tilde{q}_{ij}$], where $\tilde{q}_{ij}$ = -1 if the ith zeroed element of $\beta^* = \begin{pmatrix} \beta_1^* \\ \beta_2^* \end{pmatrix}$ has index j, and $\tilde{q}_{ij}$ = 0 otherwise.

Thus, to implement our "Style Chow Test" for testing whether or not fund style shifted at a specified time index τT, we proceed as follows.

1) Compute the Style Chow Test statistic $\mathcal{C}_T(\tau) = T(\tilde{\beta}_{1T} - \tilde{\beta}_{2T})'\Gamma^{-1}(\tilde{\beta}_{1T} - \tilde{\beta}_{2T})$, where $\tilde{\beta}_{1T}$ and $\tilde{\beta}_{2T}$ solve (2.1).
2) Run an unconstrained (e.g., OLS) version of (2.1) to identify elements of β^* to be set to zero, and form $\tilde{Q}$ accordingly.
3) Construct a large number (N) of Monte Carlo draws for $\tilde{G}_T$ based on the results of step 1).
4) Solve (2.3) for the Monte Carlo draws of step 3) using $\tilde{Q}$ as identified in step 2).
5) Obtain the *p*-value for the Style Chow Test as the area to the right of the Style Chow Test statistic $\mathcal{C}_T(\tau)$ in the Monte Carlo distribution constructed in step 4).

3. Style Shifts at an Unknown Point

In application, we often lack prior knowledge of the exact break location. Instead, we may only know that the break might have occurred within a specific time interval, say $[\tau_1 T, \tau_2 T]$, where τ_1 is the earliest time index at which a break may have occurred, and τ_2 is the latest.

In this situation, we propose using the "Maximum Style Chow Test" defined as

$$\bar{\mathcal{C}}_T \equiv \max_{\tau \in [\tau_1, \tau_2]} \mathcal{C}_T(\tau),$$

where $\mathcal{C}_T(\tau)$ is as defined in the previous section. The asymptotic distribution of $\bar{\mathcal{C}}_T$ is approximated by a Monte Carlo procedure that nests steps 1) - 4) in a loop with τ running from τ_1 through τ_2 to produce a $(\tau_2 - \tau_1 + 1) \times N$ matrix of $\tilde{\lambda}_T$ values solving (2.3) with elements $\tilde{\lambda}_T(\tau, i)$, $\tau = \tau_1, \ldots, \tau_2$; $i = 1, \ldots, N$. From these, form the statistics

$$\tilde{\omega}_T(\tau, i) = \tilde{\lambda}_T(\tau, i)' R' \Gamma(\tau)^{-1} R \tilde{\lambda}_T(\tau, i),$$

where the possible dependence of Γ on τ has been made explicit. Next, for each $i = 1, \ldots, N$, compute

$$\tilde{\omega}_T^*(i) \equiv \max_{\tau \in \{\tau_1, \ldots, \tau_2\}} \tilde{\omega}_T(\tau, i) .$$

Finally, obtain the *p*-value for the Maximum Style Chow Test as the area to the right of $\overline{\mathcal{C}}_T$ in the distribution of the $\tilde{\omega}_T^*(i)$.

4. Testing for Multiple Shifts

In practice, fund style shifts may occur at several points in time, not just once. For multiple shifts in style, one may do not know when or how often shifts may have occurred. We discuss these possibilities in this section.

The first case, multiple shifts at known points, is a straightforward extension of the single known shift point case of Section 2. To illustrate, suppose shifts are suspected to have occurred at periods τ_1 and τ_2. The style regression is

$$y_t = \tilde{X}'_{1t}\beta_1^* + \tilde{X}'_{2t}\beta_2^* + \tilde{X}'_{3t}\beta_3^* + \varepsilon_t ,$$

where $\tilde{X}_{1t}$ is a $k \times 1$ vector containing the style indices for $t = 1, \ldots, \tau_1 T$, and zero otherwise; $\tilde{X}_{2t}$ is a $k \times 1$ vector containing the style indices for $t = \tau_1 T + 1, \ldots, \tau_2 T$, and zero otherwise; and $\tilde{X}_{3t}$ is a $k \times 1$ vector containing the style indices for $t = \tau_2 T + 1, \ldots, T$, and zero otherwise. The null of no style shift is that $\beta_1^* = \beta_2^* = \beta_3^*$, which can be expressed as

$$H_0 \colon S_2 \beta^* = 0 ,$$

where $\beta^* = \begin{pmatrix} \beta_1^* \\ \beta_2^* \\ \beta_3^* \end{pmatrix}$ and $\underset{2k \times 3k}{S_2} = \begin{bmatrix} I_k & -I_k & 0 \\ 0 & I_k & -I_k \end{bmatrix}$. The solution to style regression estimate $\tilde{\beta}_T$ is obtained from the following constrained regression:

$$\min_{\beta_1, \beta_2, \beta_3} \; T^{-1} \sum_{t=1}^{T} (y_t - \tilde{X}'_{1t}\beta_1 - \tilde{X}'_{2t}\beta_2 - \tilde{X}'_{3t}\beta_3)^2 ,$$

$$
\begin{aligned}
s.t.\quad & \iota'\beta_1 = 1, \quad \beta_1 \geq 0, \\
& \iota'\beta_2 = 1, \quad \beta_2 \geq 0, \\
& \iota'\beta_3 = 1, \quad \beta_3 \geq 0.
\end{aligned}
$$

The Style Chow Statistic is given by

$$\mathcal{C}_T(\tau_1, \tau_2) = T(S_2\tilde{\beta}_T)'\Gamma^{-1}S_2\tilde{\beta}_T.$$

Critical values for $\mathcal{C}_T(\tau_1, \tau_2)$ are generated in a manner analogous to those for the single shift case, that is, from the histogram of $T(S_2\tilde{\lambda}_T)'\Gamma^{-1}S_2\tilde{\lambda}_T$, where $\tilde{\lambda}_T$ solves

$$
\begin{aligned}
& \min_{\lambda} \; (\lambda - \tilde{Z}_T)'\tilde{M}_T(\lambda - \tilde{Z}_T), \\
& s.t. \;\; \iota'\lambda_1 = 0,\; \iota'\lambda_2 = 0,\; \iota'\lambda_3 = 0,\; \tilde{Q}\lambda \leq 0,
\end{aligned}
$$

where $\lambda = (\lambda_1', \lambda_2', \lambda_3')'$ and $\tilde{M}_T$, $\tilde{Z}_T$, and $\tilde{Q}$ are constructed in the obvious way.

As with any test of structural shifts, we note that a statistically significant result may be observed in the presence of shifts that do not occur at the specified points. Rejecting the null of no shift against the alternative of a shift at specific points does not justify the conclusion that shifts did indeed occur at the hypothesized shift points. In addition, note that when testing two shifts, a statistically significant result may be observed even if there is in fact only one shift. There need not be two, and the $\mathcal{C}_T$ statistic does not tell us which suspected shift (if either) is responsible for rejecting the null.

Because prior knowledge of the style shift location is often vague, it is helpful to have procedures that can identify one or more style shift locations when these are unknown. Bai (1997) proposed a useful procedure for the standard case (no inequality restriction) that can be readily adapted to the present case. Bai's multiple shift identification procedure has the following steps.

1) Identify the first apparent shift as occurring where the Chow Statistic is highest.
2) Test whether the shift is "real" by comparing the Maximum Chow Statistic to its critical value.
3) If the shift is not statistically significant, stop.
4) If the shift is significant, split the sample into two sub-samples: before and after the shift.

5) Apply the Maximum Chow Test within each sub-sample to identify further shifts – that is, repeat steps 1) - 4) in each sub-sample.
6) Continue until no sub-sample Maximum Chow Statistic is significant.

To adapt Bai's procedure to the present contest, we simply replace the standard Maximum Chow Statistic with the Maximum Style Chow Statistic.

5. Empirical Illustration

Here, we apply the proposed method to real data to demonstrate how structural breaks in manager style can be detected. Specifically, we show how the management style in the Fidelity Magellen Fund (FMF) was structurally changed. The fund is US-domiciled and well known for its active management. The sample period is January 1988 through December 2017 and includes 360 monthly observations. We use the Russell indices (Russell 2000 growth, Russell 2000 value, Russell 1000 growth, and Russell 1000 value) as explanatory variables. Table 1 shows relevant summary statistics of the variables.[2]

Some relevant break statistics from our procedure when applied to the whole sample period (1988.01 - 2017.12) are shown in Table 2. We set the number of Monte Carlo replications for linear-quadratic optimization to be 1,000 in all computations. To avoid instability in estimation, we ensure that the number of observations in both initial and final estimation windows is at least 30; that is, $\tau_1 T = 30$ and $(1-\tau_2)T = 30$. The Maximum Style Chow statistic obtained from the whole sample period is 17.38 with *p*-value close to zero (0.002) and the estimated break date is March 1992, suggesting that there are two style regimes before and after the break date. How the management style shifted between these two regimes is shown in Table 2, under the headings "Regime 1" and "Regime 2." The results indicate the clear shift into large cap, away from heavy value orientation. During the first regime, no weight is given to growth stocks, and all funds are invested on small cap value (47%) and large cap value (53%). However, a noticeable change occurs around the break date; all funds are withdrawn from small cap stocks and distributed over large cap growth (60%) and large cap value (40%).

Since the break (March 1992) is statistically significant, we can split the sample into two sub-samples to search for possible multiple breaks. However, the resulting first sub-sample (1988.01 –

[2] All data supporting the findings of this study have been downloaded from Yahoo Finance (https://finance.yahoo.com/quote/FMAGX/history?p=FMAGX) and are also available from the corresponding author (T.-H. Kim) upon request.

1992.03 with 51 observations) is too small to be subject to the grid search for possible additional breaks. Hence, we do not analyze it further. In contrast, the second sub-sample (1992.04 - 2017.12 with 309 observations) is sufficiently long to compute the relevant break statistics.

The estimation results are shown in Table 3. The Maximum Style Chow statistic is 10.06 with *p*-value close to zero (0.019) and the estimated break date is October 2002, which indicates the presence of two further regimes (denoted "Regime 2-1" for the sub-period of 1992.04 - 2002.10 and "Regime 2-2" for 2002.11 - 2017.12). The estimated style weights show a clear shift into growth stocks away from large cap orientation (50% on Russell 1000 Growth and 50% on Russell 1000 Value). A rather noticeably large weight (87%) is placed on Russell 1000 growth during Regime 2-2.

The sub-regimes, Regime 2-1 and Regime 2-2, have 127 and 182 observations, respectively. Hence, we further applied the proposed method to each sub-regime in search of additional breaks. However, no more structural breaks in style are found in both sub-regimes.

6. Conclusion

A fund manager's style can change, shifting from one position to another at a point of time. No formal method for testing whether changes in manager style has existed when the standard two restrictions (sum-to-one and non-negativity) are jointly imposed in style analysis. Combining the well-known Chow test and the results of Andrews (1997a, 1997b, 1999, 2000), we developed a formal test for shifts in manager style. Our testing procedure is general in that the potential change-point is not assumed to be known, and the case of multiple shifts is entertained. Application of our proposed test to the FMF revealed that the fund's style changed at least twice between 1988 and 2017.

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Table 1. Summary Statistics for Fidelity Magellan Fund (percent return)

Statistics	Sample period: 1988.01 - 2017.12 (360 monthly observations)				
	fidelity	r2growth	r2value	r1growth	r1value
Mean	0.83	0.66	0.73	0.70	0.62
Median	1.15	1.44	1.36	1.12	1.10
Maximum	20.69	21.71	19.43	16.52	15.38
Minimum	-26.43	-24.49	-26.53	-18.64	-20.12
Std. Dev.	5.37	6.53	5.29	4.83	4.34
Skewness	-0.31	-0.62	-0.99	-0.71	-0.96
Kurtosis	3.66	1.43	3.88	2.14	3.76
Correlation					
fidelity	1.00				
r2growth	0.65	1.00			
r2value	0.62	0.86	1.00		
r1growth	0.73	0.84	0.72	1.00	
r1value	0.70	0.72	0.86	0.81	1.00

Table 2. Break Statistics with Fidelity Magellan Fund

	Whole Sample Period: 1988.01 - 2017.12 (360 observations)
Break Date	1992.03
Max Style Chow Statistic	17.38
P-Value	0.002
Style Regression	
Regime 1	1988.01 - 1992.03 (51 observations)
Russell 2000 Growth	0.00 (0.14)
Russell 2000 Value	0.47 (0.16)
Russell 1000 Growth	0.00 (0.05)
Russell 1000 Value	0.53 (0.06)
Regime 2	1992.04 - 2017.12 (309 observations)
Russell 2000 Growth	0.00 (0.03)
Russell 2000 Value	0.00 (0.03)
Russell 1000 Growth	0.60 (0.07)
Russell 1000 Value	0.40 (0.07)

Note that standard errors are parentheses.

Table 3. Multiple Break Statistics with Fidelity Magellan Fund

(whole sample period: 1986.10 – 2014.01)

	Sample Period subject to Search: 1992.04 - 2017.12 (309 observations)
Break Date	2002.10
Max Style Chow Statistic	10.06
P-Value	0.019
Style Regression	
Regime 2-1	1992.04 - 2002.10 (127 observations)
Russell 2000 Growth	0.00 (0.03)
Russell 2000 Value	0.00 (0.04)
Russell 1000 Growth	0.50 (0.07)
Russell 1000 Value	0.50 (0.07)
Regime 2-2	2002.11 - 2017.12 (182 observations)
Russell 2000 Growth	0.13 (0.19)
Russell 2000 Value	0.00 (0.17)
Russell 1000 Growth	0.87 (0.19)
Russell 1000 Value	0.00 (0.18)

Note that standard errors are parentheses.